Unit 4

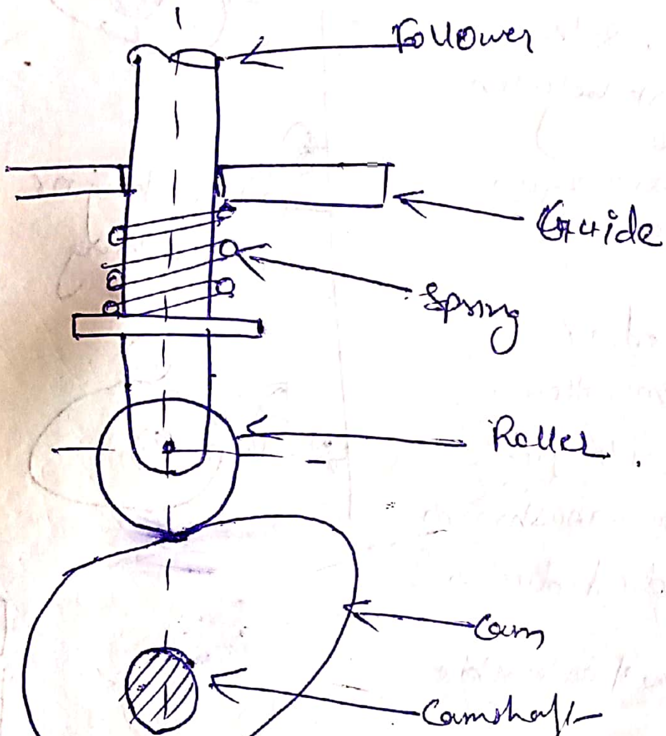
①

CAM & FOLLOWERS

Cam is a rotating machine element which gives reciprocating or oscillating motion to another element called follower.

They have line contact hence higher pair

Used in I.C. Engine valves, paper cutting, printing machines automatic lathes.



Spring is used to maintain direct contact

Cam follower is a 3 link mechanism of higher pair.

① Cam

② Follower

③ Frame

Followers

[On Basis of Contact Surface]

1. Knife edge
   (small area of contact results excessive wear) rarely used.

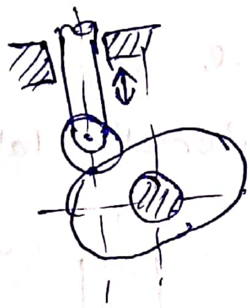
2. Roller Follower (widely used)
   (Rate of wear is reduced, side thrust exist between guide & Follower)
   - used in gas & oil engines
   - aircraft engines

3. Flat faced or mushroom follower.
   (a) Flat surface - flat faced
   (b) Circular Surface - mushroom
   - side thrust reduced due to friction
   - used in valves of automobile

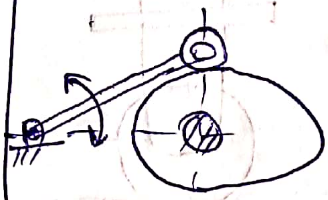
4. Spherical faced
   Flat ~~end~~ faced follower ~~...~~ ~~...~~ induces high surface stress hence to reduce it spherical faced followers are used

[Type of motion]

1. Reciprocating or Translating

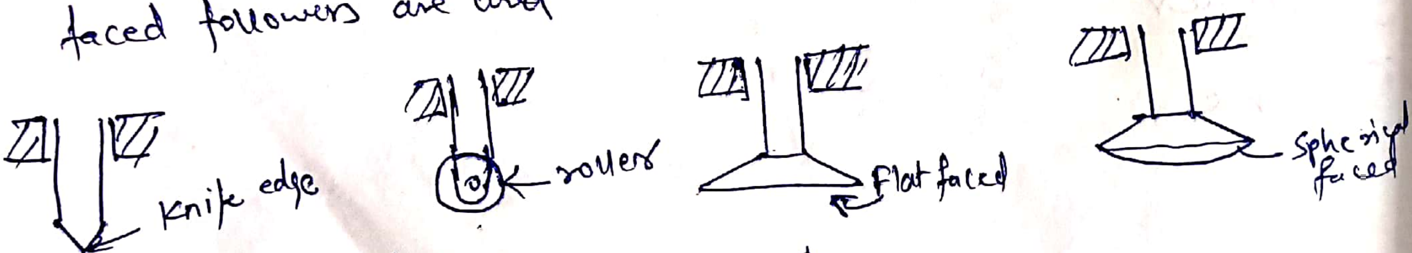


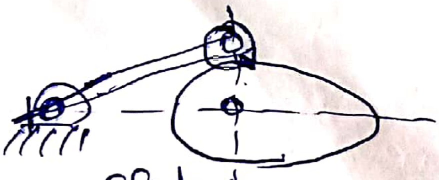
2. Rotating or oscillating



[Path of motion]

1. Radial follower
2. Offset follower.



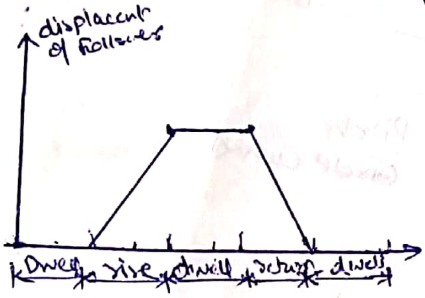


* Classification of Cams

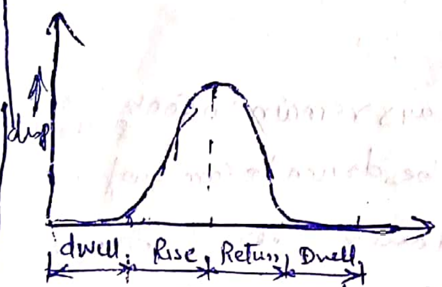
Cams

[According to Follower motion]

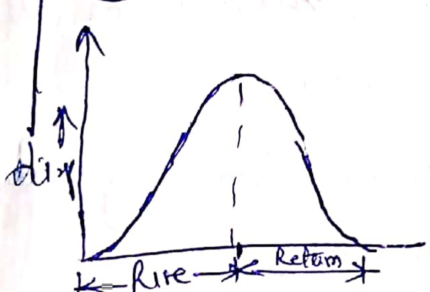
① DRDRD Cam (widely used)



② DRRD Cam



③ RRR Cam (rarely used)

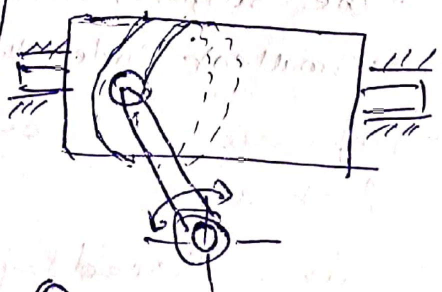


[According to Shape of Cam]

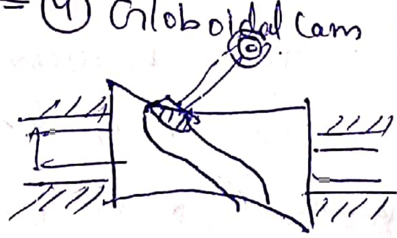
① Translation or Flat or wedge Cam

② Radial or Disc Cam

③ Cylindrical or drum cam



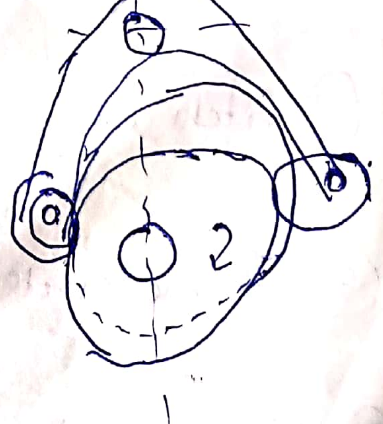
④ Globoidal Cam



[Similar to cylindrical Cam but only concave or convex globoids are used.

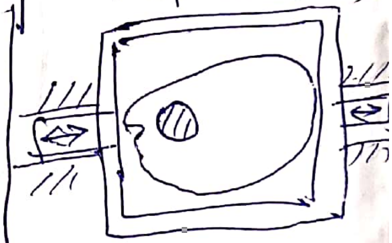
⑤ Conjugate or double disc Cam

used in high speed and high dynamic load applicat.



[According to manner of Constraint]

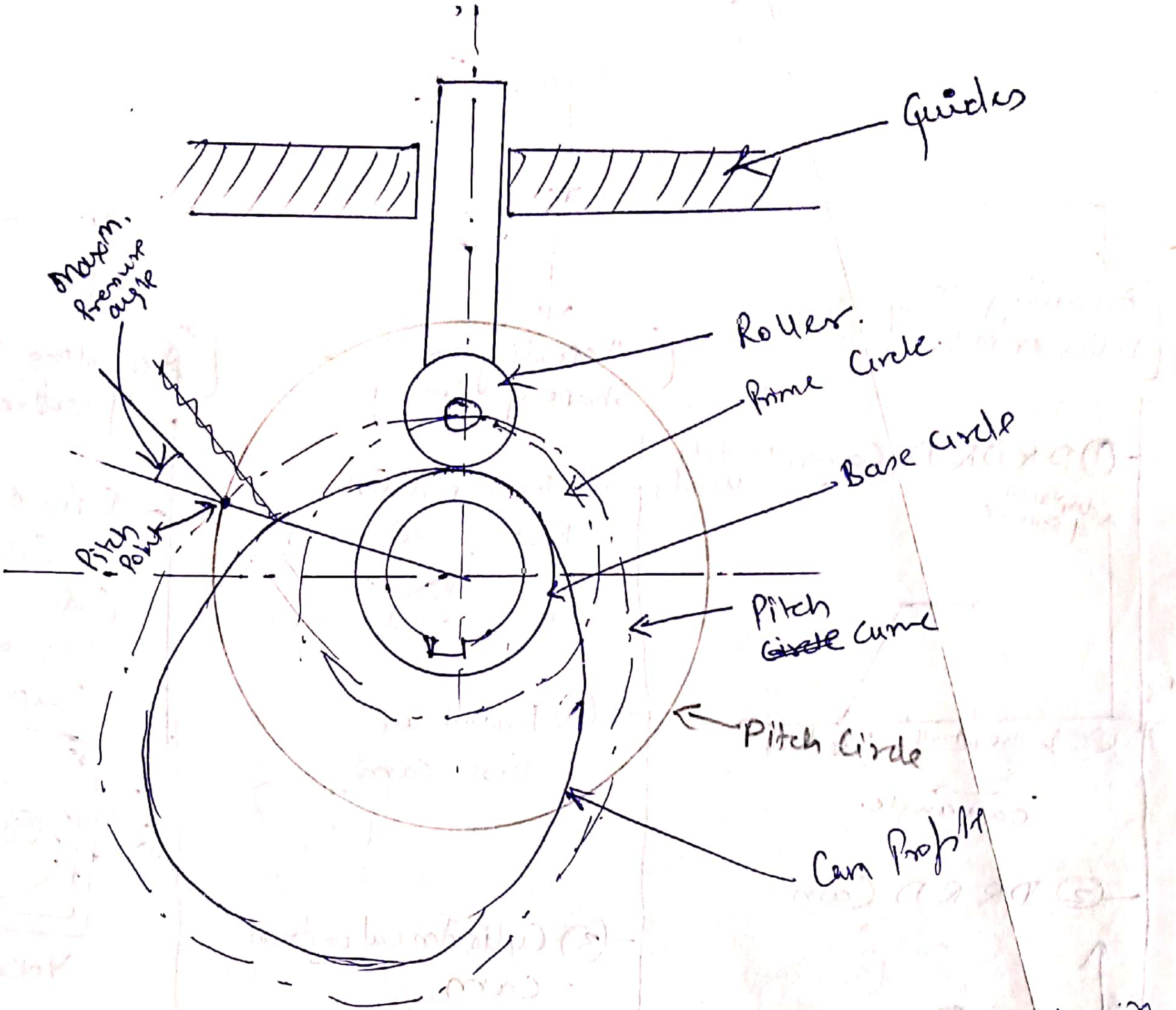
① Positive drive Cam (Yoke cam) (doesn't require any support to keep contact)



Yoke Cam

② Preloaded Spring Cam.

Basic Terminology



1. Cam profile – The surface which always remain in contact with follower
2. Base circle – smallest circle that can be drawn to cam profile.
3. Trace Point – Reference Point on follower used to generate Pitch curve
4. Pitch curve – Curve traced by trace point
5. Pressure angle :- angle between axis of motion of follower & normal to pitch curve.
6. Pitch point : ~~Pitch~~ Point on pitch curve having maxm Pressure angle
7. Pitch circle : Circle drawn from pitch point ~~thro~~ by taking cam centre
8. Prime circle : smallest circle drawn through cam centre & tangent to pitch curve.
9. Lift or stroke – maxm displacement of follower from base circle

(10) Cam angle = angle of rotation of cam for particular displacement of follower. (3)

(11) Angle of ascent – Angle turned by cam when follower rises

(12) Angle of descent – " " " " " " returns

(13) Angle of dwell – " " " " " dwells

(14) Angle of Action – Total angle turned by cam during the time between the start of rise & return of follower.

* Types of Motion of follower

during operation, Cam → uniform velocity but follower may have

1. Uniform velocity
2. SHM
3. Uniform acceleration and ~~deceleration~~ retardation
4. Cycloidal Motion.

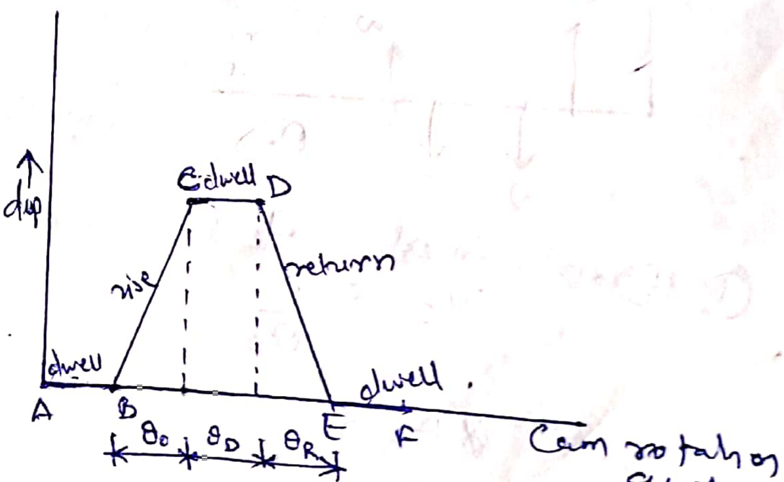
(1) Motion of follower with uniform velocity.

$$\therefore \quad v = \frac{ds}{dt}$$

$$a = \frac{dv}{dt}$$

and jerk, $j = \frac{da}{dt}$

As follower moves with uniform velocity during its rise & return stroke, slope of displacement curve must be constant



Analytical solution :–

S = maximum follower displacement.

Y = Instantaneous follower displacement

θ_0 = Cam rotation for max^m follower displ.

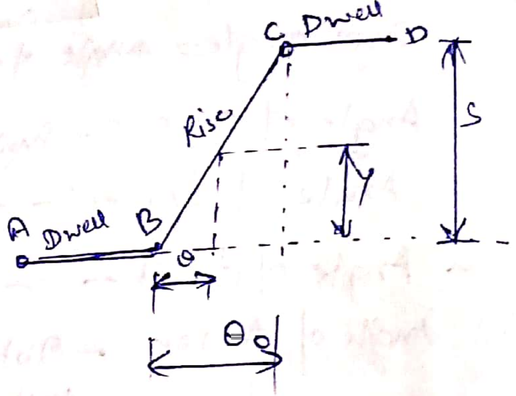
θ = Instantaneous cam rotation angle

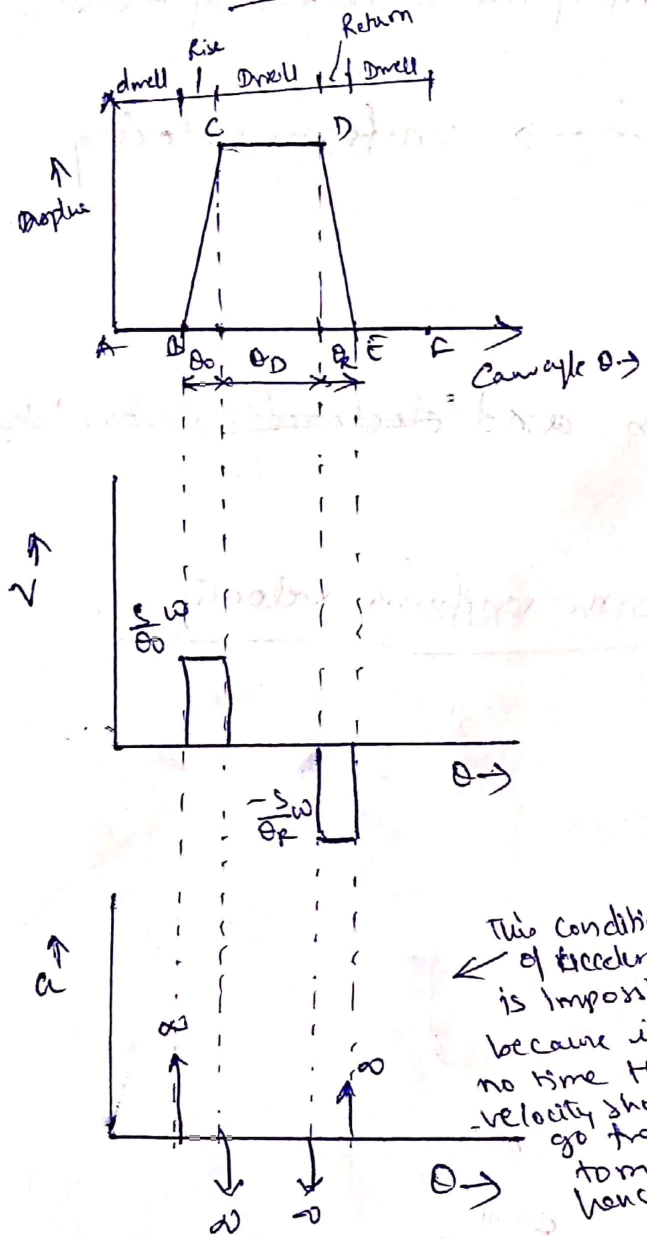
* Basic Terminology

Similar triangle rule.

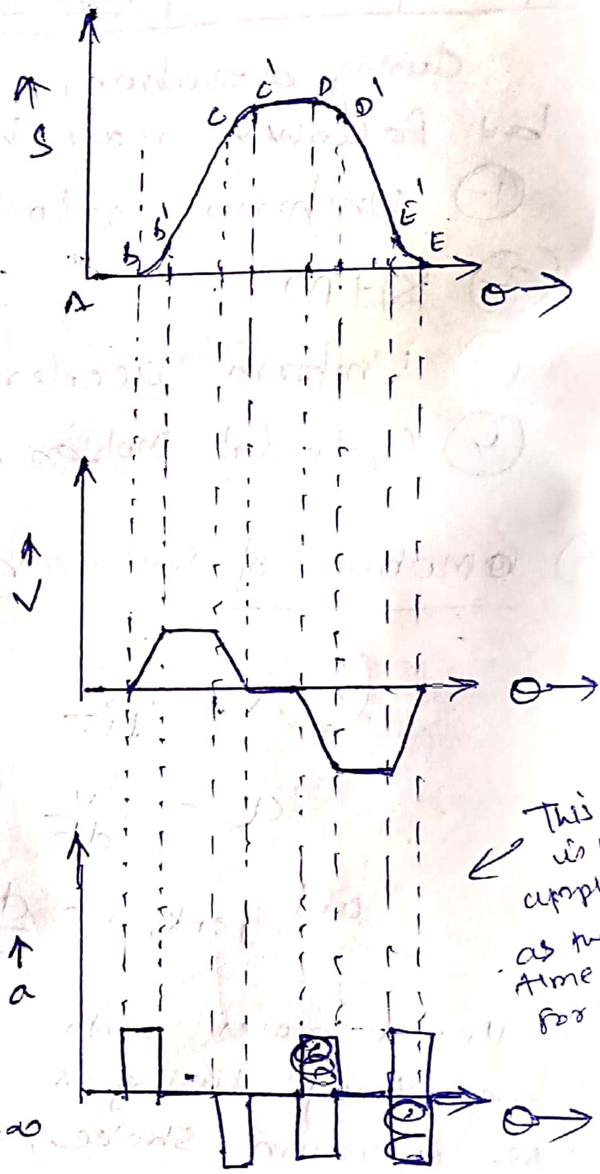
$$\frac{y}{\theta} = \frac{S}{\theta_0}$$

$$\boxed{y = \frac{S \cdot \theta}{\theta_0}}$$





This condition of acceleration is impossible because in no time the velocity should go from 0 to max or hence a = ∞



This condition is partially applicable as there is time given for acceleration

① Displacement of follower at $\theta = 0$, $\theta = \frac{\theta_0}{2}$, $\theta = \theta_0$

$$y = S \times \frac{\theta}{\theta_0}$$

at $\theta = 0$,

$$y = 0$$

at $\theta = \frac{\theta_0}{2}$

$$y_{\frac{\theta_0}{2}} = S \times \frac{\theta_0/2}{\theta_0} = \frac{S}{2}$$

at $\theta = \theta_0$

C(4)

$$Y_{\theta=\theta_0} = S \times \frac{\theta_0}{\theta_0}$$

$$\boxed{Y = S}$$ —— Displacement of follower.

(2) velocity of Follower

$$V = \frac{dy}{dt} = \frac{d}{dt}\left(S \times \frac{\theta}{\theta_0}\right)$$

$$V = \frac{S}{\theta_0} \cdot \frac{d\theta}{dt}$$

$$\boxed{V = \frac{\omega \cdot S}{\theta_0}}$$ —— as velocity is independent of θ

So,

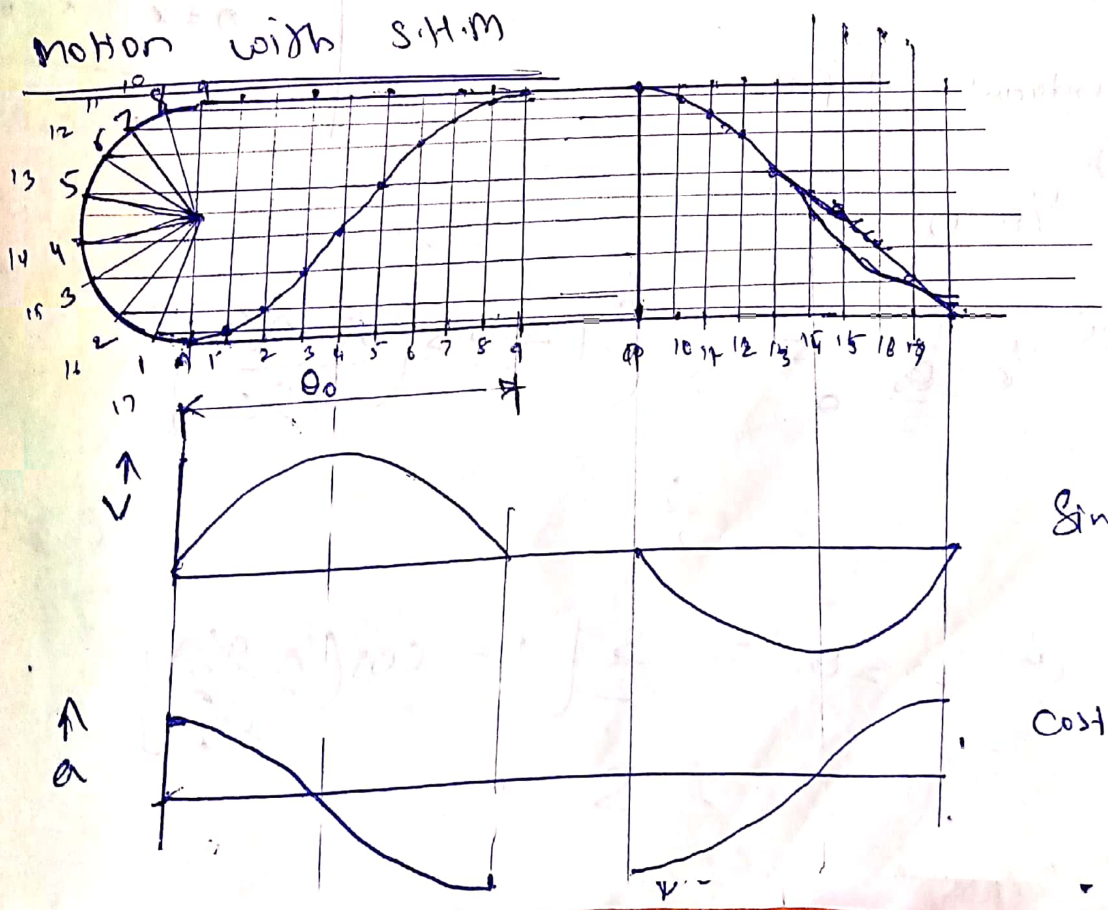
$$V_0 = V_{\frac{\theta_0}{2}} = V_{\theta_0} = \frac{S}{\theta_0}\omega$$

(3) Accelration of Follower:

$$a = \frac{dv}{dt} = \frac{d}{dt}\left(\frac{S}{\theta_0}\omega\right) = 0$$

Acceleration is zero for complete stroke. But at start & end stroke follower is required to attain velocity in min. time hence acceleration is infinite

* Motion with S.H.M



Sine curve

Cosine curve.

Analytical approach

$$\cancel{x_\theta}\ Y = \cancel{QP'} = \cancel{OQ - OP'}$$

$$\cancel{Y = \frac{S}{2} -}$$

when $\theta_0 = \pi$ for outstroke angle

Similarly at certain angle θ, the angle of rotation will be.

$$\frac{x}{\theta} = \frac{\pi}{\theta_0}$$

$$x = \frac{\pi \cdot \theta}{\theta_0}$$

$$Y = OQ - OP'$$

$$= \frac{S}{2} - \frac{S}{2}\cos\left(\frac{\pi\theta}{\theta_0}\right)$$

$$\boxed{Y = \frac{S}{2}\left[1 - \cos\left(\frac{\pi\theta}{\theta_0}\right)\right]}$$

displacement of follower.

① at $Y_{\theta=0} = 0$

② at $Y_{\theta=\frac{\theta_0}{2}} = \frac{S}{2}\left[1 - \cos\left(\frac{\pi\cancel{\theta_0}}{2\cancel{\theta_0}}\right)\right]$

$$= \frac{S}{2}$$

③ at $Y_{\theta=\theta_0} = \frac{S}{2}\left[1 - \cos\left(\frac{\pi\cancel{\theta_0}}{\cancel{\theta_0}}\right)\right]$

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velocity of follower,

$$v = \frac{dy}{dt} = \frac{d}{dt}\left[\frac{S}{2}\left(1-\cos\left(\frac{\pi\theta}{\theta_0}\right)\right)\right]$$

$$\frac{dy}{dt} = \frac{dy}{d\theta} \times \frac{d\theta}{dt}$$

$$= \omega \cdot \frac{dy}{d\theta}$$

$$= \omega \frac{d}{d\theta}\left[\frac{S}{2}\left(1-\cos\left(\frac{\pi\theta}{\theta_0}\right)\right)\right]$$

$$= \omega\left[\frac{S}{2}\left(+\sin\left(\frac{\pi\theta}{\theta_0}\right)\cdot\frac{\pi}{\theta_0}\right)\right]$$

$$\boxed{V = \left(\frac{\pi S \omega}{2\theta_0}\right)\left[\sin\left(\frac{\pi\theta}{\theta_0}\right)\right]}$$

$$V_{\theta=0} = \frac{\pi S\omega}{2\theta_0}(0) = 0$$

$$V_{\left(\theta=\frac{\theta_0}{2}\right)} = \left(\frac{\pi S\omega}{2\theta_0}\right)\sin\left(\frac{\pi\cancel{\theta_0}}{2\cancel{\theta_0}}\right)$$

$$= \frac{\pi S\omega}{2\theta_0}$$

$$V_{(\theta=\theta_0)} = \left(\frac{\pi S\omega}{2\theta_0}\right)\sin\left(\frac{\pi\cancel{\theta_0}}{\cancel{\theta_0}}\right)$$

$$= 0$$

It means maximum velocity is at $\theta = \frac{\theta_0}{2}$

$$\boxed{V_{max} = \frac{\pi S\omega}{2\theta_0}}$$

Acceleration of follower

$$a = \frac{dv}{dt} \Rightarrow \frac{dv}{d\theta} \times \frac{d\theta}{dt} \Rightarrow \omega \cdot \frac{dv}{d\theta}$$

$$a = \omega \cdot \frac{d}{d\theta}\left[\frac{\pi S \omega}{2\theta_0} \sin\left(\frac{\pi\theta}{\theta_0}\right)\right]$$

$$= \omega \cdot \left[\frac{\pi S \omega}{2\theta_0} \cos\left(\frac{\pi\theta}{\theta_0}\right) \cdot \left(\frac{\pi}{\theta_0}\right)\right]$$

$$\boxed{a = \frac{\pi^2 S \omega^2}{2\theta_0^2}\left[\cos\left(\frac{\pi\theta}{\theta_0}\right)\right]}$$

$$a_{(\theta=0)} = a_0 = \frac{\pi^2 S \omega^2}{2\theta_0^2}$$

$$a\left(\theta = \frac{\theta_0}{2}\right) = \frac{\pi^2 S\omega^2}{2\theta_0^2}\left[\cos\left(\frac{\pi \cancel{\theta_0}}{2\cancel{\theta_0}}\right)\right] = 0$$

$$a(\theta = \theta_0) = \frac{\pi^2 S \omega^2}{2\theta_0^2} \cos\left(\frac{\pi \cancel{\theta_0}}{\cancel{\theta_0}}\right) = -\frac{\pi^2 S\omega^2}{2\theta_0^2}$$

So,

$$\boxed{a_{max} = \pm \frac{\pi^2 S \omega^2}{2\theta_0^2}}$$

Jerk in follower

$$j = \frac{da}{dt} \Rightarrow \frac{da}{d\theta} \times \frac{d\theta}{dt} \Rightarrow \omega \cdot \frac{da}{d\theta}$$

$$j = \omega \cdot \frac{d}{d\theta}\left[\frac{\pi^2 S \omega^2}{2\theta_0^2}\left\{\cos\left(\frac{\pi\theta}{\theta_0}\right)\right\}\right]$$

$$= \omega \cdot \frac{\pi^2 S\omega^2}{2\theta_0^2}\left(-\sin\frac{\pi\theta}{\theta_0}\right) \cdot \left(\frac{\pi}{\theta_0}\right)$$

$$\boxed{j = -\frac{\pi^3 S \omega^3}{2\theta_0^3}\left[\sin\left(\frac{\pi\theta}{\theta_0}\right)\right]}$$

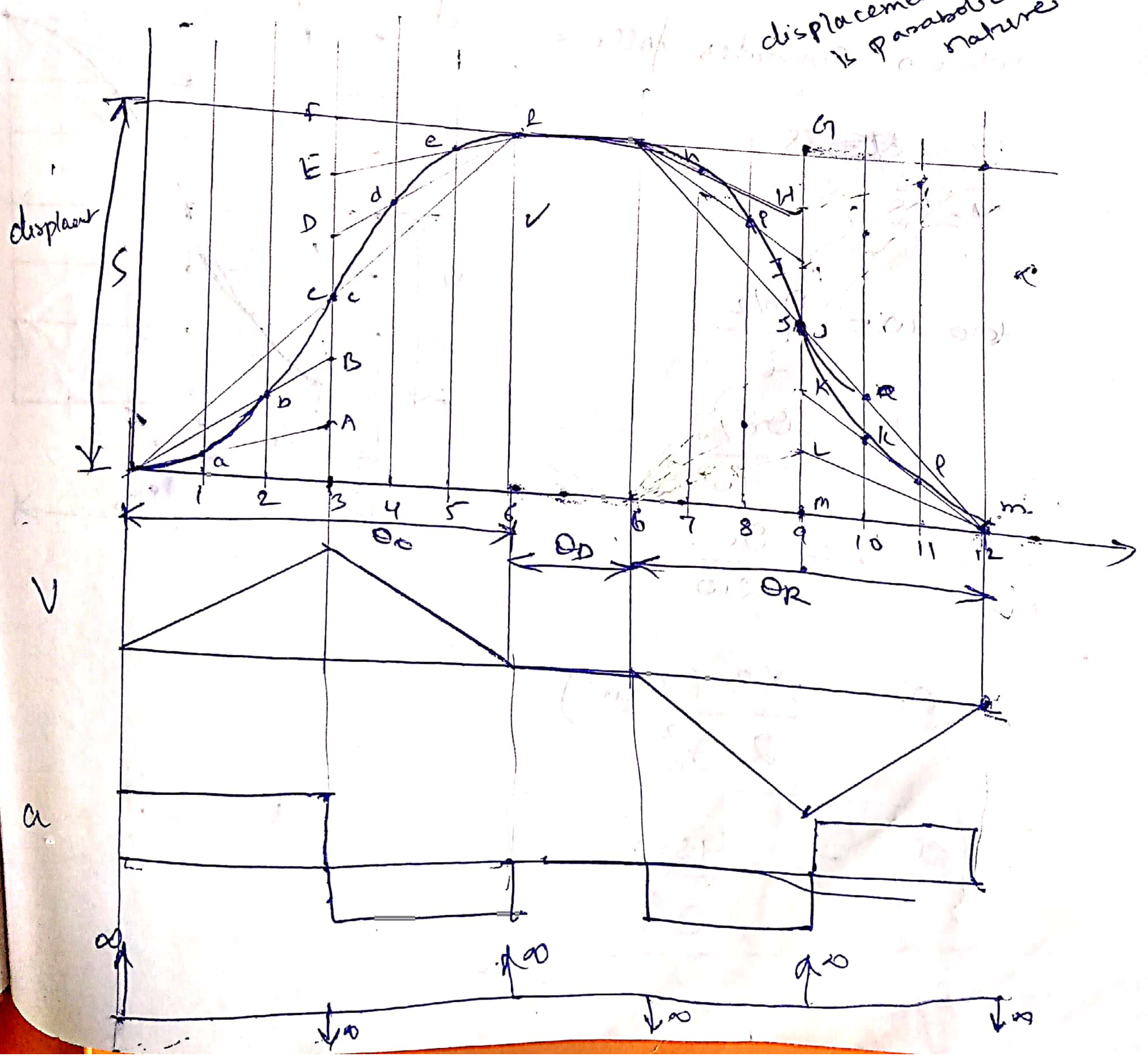
$$j_{\theta=0} = j_0 = -\frac{\pi^3 s\omega^3}{2\theta_o^3}\sin\left(\frac{\pi(0)}{\theta_o}\right) = 0$$

$$j_{\theta=\frac{\theta_o}{2}} = j_{\frac{\theta_o}{2}} = -\frac{\pi^3 s\omega^3}{2\theta_o^3}\cdot\sin\left(\frac{\pi\cancel{\theta_o}}{2\cancel{\theta_o}}\right) = -\frac{\pi^3 s\omega^3}{2\theta_o^3}$$

$$j_{\theta=\theta_o} = j_{\theta_o} = -\frac{\pi^3 s\omega^3}{2\theta_o^3}\sin\left(\frac{\pi\cancel{\theta_o}}{\cancel{\theta_o}}\right) = 0.$$

$$\boxed{j_{max} = -\frac{\pi^3 s\omega^3}{2\theta_o^3}}$$

Motion of follower with Uniform Acceleration & Retardation



Analytical Solution

For uniform acceleration displacement

$$S = ut + \frac{1}{2}at^2$$

Similarly

$$y = V_0 t + \frac{1}{2}at^2.$$

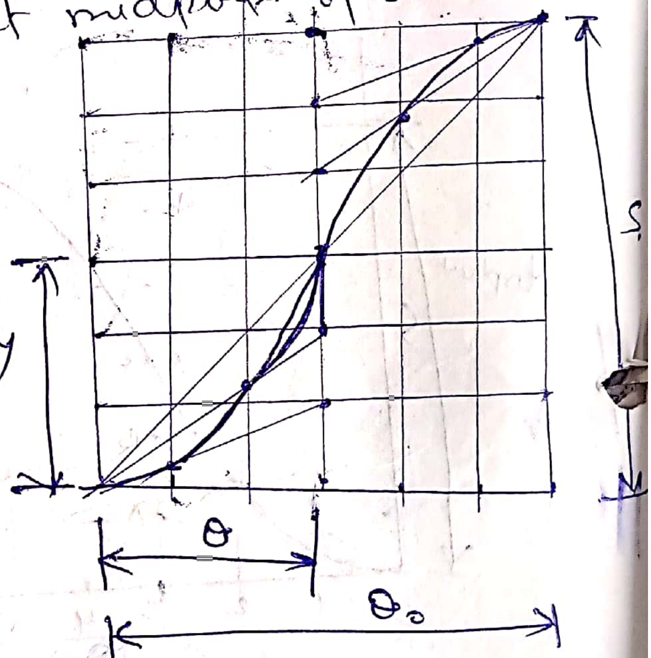
$V_0 = 0$ = Initial velocity of follower

so, $y = \frac{1}{2}at^2$

$$\boxed{a = \frac{2y}{t^2}.}$$

as acceleration is constt. during accelerating period.

Let us consider follower at midpoint of outstroke



$$y = \frac{S}{2}$$

$$\omega = \frac{\theta_0/2}{t}$$

$$t = \frac{\theta_0/2}{\omega}$$

$$t = \frac{\theta_0}{2\omega}$$

$$a = \frac{\cancel{2}S(4\omega^2)}{\cancel{2}\theta_0^2}$$

$$\boxed{a = \frac{4S\omega^2}{\theta_0^2}}$$

<u>displacement</u>

$$\left.\begin{aligned} Y_{\theta=0} &= 0 \\ Y_{\theta=\frac{\theta_0}{2}} &= \frac{S}{2} \\ Y_{\theta=\theta_0} &= S. \end{aligned}\right\} \text{From fig.}$$

<u>Velocity</u>.

$$V = \frac{dy}{dt} = \frac{d}{dt}\left(\frac{1}{2}at^2\right)$$

$$V = \frac{1}{2}a(2t)$$

$$V = at$$

$$V = \frac{4S\omega^2}{\theta_0^2} \cdot t$$

$$V = \frac{4S\omega^2}{\theta_0^2} \cdot \frac{\theta}{\omega}$$

$$\boxed{V = \frac{4S\omega}{\theta_0^2} \cdot \theta.}$$

$$V_{\theta=0} = 0$$

$$V_{\theta=\frac{\theta_0}{2}} = \frac{4S\omega}{\theta_0^2} \cdot \frac{\theta_0}{2} = \frac{2S\omega}{\theta_0}$$

$$V_{\theta=\theta_0} = 0 \cancel{\frac{4S\omega}{\theta_0^2} \cdot \theta_0 = \frac{4S\omega}{\theta_0}}$$

Practically, In this case,

the motion is uniform retardation so the velocity reduces to zero.

Acceleration

$$a = \frac{4 S \omega^2}{\theta_0^2}$$

As acceleration is not function of θ, its magnitude is first & second half remains same. value of ~~accel~~ acceleration is same for complete stroke.

In second half direction of acceleration is opposite to first half.

Jerk

$$J_{\theta=0} = j_0 = \infty$$

$$J_{\theta = \frac{\theta_0}{2}} = \infty$$

$$J_{\theta=\theta_0} = \infty$$

Because in no time the acceleration is increasing. $J = \frac{a}{t} = \frac{a}{0} = \infty$

Infinite jerk will produce shocks & vibration hence suitable ~~for~~ for low & moderate cam speed applications.

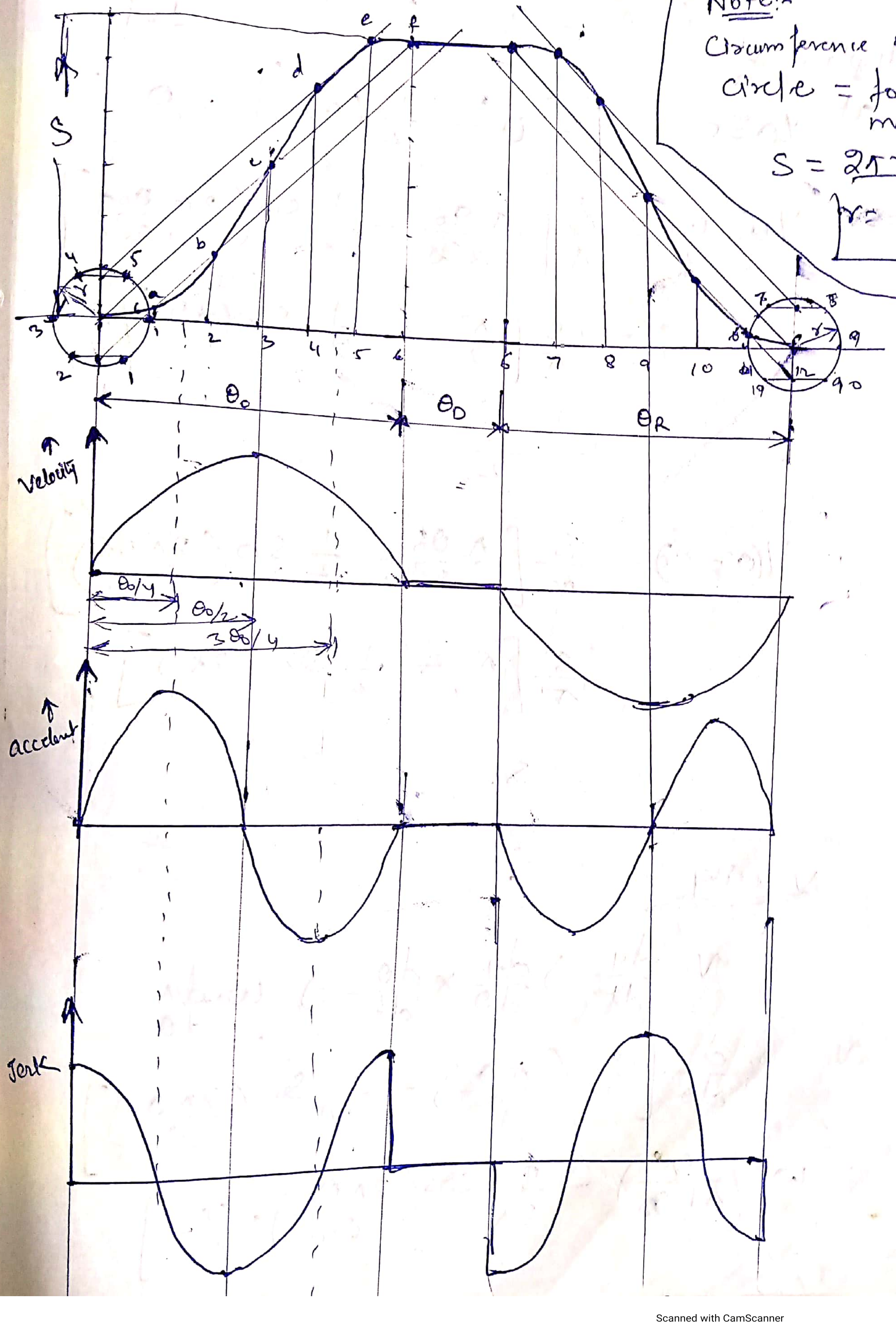
* Motion of Follower with cycloidal motion

Cycloid → Curve traced by a point on circle when circle rolls without sliping on straight line.

Note:-
Circumference of circle = follower max^m displ.

$S = 2\pi r$

$\boxed{r = \frac{S}{2\pi}}$



Analytical

mathematical cycloidal motion is given by,

$$\boxed{y = \frac{s}{\pi}\left[\frac{\pi\theta}{\theta_0} - \frac{1}{2}\sin\left(\frac{2\pi\theta}{\theta_0}\right)\right]}$$

displacement

at $y_{\theta=0} = \frac{s}{\pi}\left[0 - \frac{1}{2}(0)\right] = 0.$

$$y_{\theta=\frac{\theta_0}{2}} = \frac{s}{\pi}\left[\frac{\pi\cancel{\theta_0}}{2\cancel{\theta_0}} - \frac{1}{2}\sin\left(\frac{2\pi\cancel{\theta_0}}{2\cancel{\theta_0}}\right)\right]$$

$$= \frac{s}{\pi}\left[\frac{\pi}{2} - \frac{1}{2}\sin(\pi)\right]$$

$$= \frac{s}{2}.$$

$$y(\theta=\theta_0) = \frac{s}{\pi}\left[\frac{\pi\cancel{\theta_0}}{\cancel{\theta_0}} - \frac{1}{2}\sin\left(\frac{2\pi\cancel{\theta_0}}{\cancel{\theta_0}}\right)\right]$$

$$= \frac{s}{\pi}\left[\pi - \frac{1}{2}\sin(2\pi)\right]$$

$$= s.$$

velocity

$$v = \frac{dy}{dt} \Rightarrow \frac{dy}{d\theta}\times\frac{d\theta}{dt} \Rightarrow \omega.\frac{dy}{d\theta}$$

$$v = \omega.\frac{d}{d\theta}\left[\frac{s}{\pi}\left(\frac{\pi\theta}{\theta_0}\right) - \frac{s}{2\pi}\sin\left(\frac{2\pi\theta}{\theta_0}\right)\right]$$

$$= \omega.\left[\frac{s}{\pi}\left(\frac{\pi}{\theta_0}\right) - \frac{s}{2\pi}\cos\left(\frac{2\pi\theta}{\theta_0}\right).\frac{2\pi}{\theta_0}\right]$$

(9)

$$V = \omega\left[\frac{S}{\pi}\left(\frac{\pi}{\theta_0}\right) - \frac{S\pi}{\theta_0\pi}\cos\left(\frac{2\pi\theta}{\theta_0}\right)\right]$$

$$= \omega\left[\frac{S}{\theta_0} - \frac{S\cancel{\pi}}{\theta_0\cancel{\pi}}\cos\left(\frac{2\pi\theta}{\theta_0}\right)\right]$$

$$\boxed{V = \frac{S\omega}{\theta_0}\left[1 - \cos\left(\frac{2\pi\theta}{\theta_0}\right)\right]}$$

$$V(\theta=0) = \frac{S\omega}{\theta_0}\left[1-1\right] = 0$$

$$V\left(\theta=\frac{\theta_0}{2}\right) = \frac{S\omega}{\theta_0}\left[1-\cos\left(\frac{\cancel{2}\pi\cancel{\theta_0}}{\cancel{2\theta_0}}\right)\right]$$

$$= \frac{2S\omega}{\theta_0}$$

$$V\left(\theta=\frac{\theta_0}{4}\right) = \frac{S\omega}{\theta_0}\left[1-\cos\left(\frac{2\pi\cancel{\theta_0}}{\cancel{2}4\cancel{\theta_0}}\right)\right]$$

$$= \frac{S\omega}{\theta_0}\left[1-0\right] = \frac{S\omega}{\theta_0}$$

$$V\left(\theta=\frac{3\theta_0}{4}\right) = \frac{S\omega}{\theta_0}\left[1-\cos\left(\frac{\cancel{2}\pi(3\cancel{\theta_0})}{{}_2\cancel{4}\cancel{\theta_0}}\right)\right]$$

$$= \frac{S\omega}{\theta_0}\left[1\right]$$

$$= \frac{S\omega}{\theta_0}$$

$$V(\theta=\theta_0) = \frac{S\omega}{\theta_0}\left(1-\cos\left(\frac{2\pi\cancel{\theta_0}}{\cancel{\theta_0}}\right)\right)$$

$$= 0$$

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$$\boxed{V_{max} = \frac{2S}{\theta_0}\omega}$$

Acceleration

$$a = \frac{dv}{dt} \Rightarrow \frac{dv}{d\theta} \times \frac{d\theta}{dt} \Rightarrow \omega \cdot \frac{dv}{d\theta}$$

$$a = \omega \cdot \frac{d}{d\theta}\left[\frac{S\omega}{\theta_0}\left\{1 - \cos\left(\frac{2\pi\theta}{\theta_0}\right)\right\}\right]$$

$$= \frac{S\omega^2}{\theta_0}\frac{d}{d\theta}\left[1 - \cos\left(\frac{2\pi\theta}{\theta_0}\right)\right]$$

$$= \frac{S\omega^2}{\theta_0}\left[+\sin\left(\frac{2\pi\theta}{\theta_0}\right) \cdot \frac{2\pi}{\theta_0}\right)$$

$$\boxed{a = \frac{2S\pi\omega^2}{\theta_0^2}\sin\left(\frac{2\pi\theta}{\theta_0}\right)}$$

$$a(\theta = 0) = \frac{2\pi S\omega^2}{\theta_0^2}(0) = 0$$

$$a\left(\theta = \frac{\theta_0}{4}\right) = \frac{2\pi S\omega^2}{\theta_0^2}\sin\left(\frac{\cancel{2}\pi \cancel{\theta_0}}{\cancel{2}4\cancel{\theta_0}}\right)$$

$$= \frac{2\pi S\omega^2}{\theta_0^2}$$

$$a\left(\theta = \frac{\theta_0}{2}\right) = \frac{2\pi S\omega^2}{\theta_0^2}\left[\sin\left(\frac{\cancel{2}\pi\cancel{\theta_0}}{\cancel{2}\cancel{\theta_0}}\right)\right]$$

$$= 0$$

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(10)

$$a\left(\theta=\frac{3\theta_0}{4}\right) = \frac{2\pi s\omega^2}{\theta_0^2}\left[\sin \frac{2\pi}{\cancel{\theta_0}}\left(\frac{3\cancel{\theta_0}}{4}\right)\right]$$

$$= \frac{2\pi s\omega^2}{\theta_0^2}\left[\sin \frac{3\pi}{2}\right]$$

$$= -\frac{2\pi s\omega^2}{\theta_0^2}$$

$$a(\theta=\theta_0) = \frac{2\pi s\omega^2}{\theta_0^2}\left[\sin\left(\frac{2\pi\cancel{\theta_0}}{\cancel{\theta_0}}\right)\right] = 0$$

Jerk

$$j = \frac{da}{dt} \Rightarrow \frac{da}{d\theta}\times\frac{d\theta}{dt} = \omega\cdot\frac{da}{d\theta}$$

$$= \omega\cdot\frac{d}{d\theta}\left[\frac{2s\pi\omega^2}{\theta_0^2}\cdot\sin\left(\frac{2\pi\theta}{\theta_0}\right)\right]$$

$$= \frac{2s\pi\omega^3}{\theta_0^2}\left[\cos\left(\frac{2\pi\theta}{\theta_0}\right)\cdot\frac{2\pi}{\theta_0}\right]$$

$$\boxed{j = \frac{4\pi^2\omega^3\cdot s}{\theta_0^3}\left[\cos\left(\frac{2\pi\theta}{\theta_0}\right)\right]}$$

$$j(\theta=0) = \frac{4\pi^2 s\omega^3}{\theta_0^3}[1] = \frac{4\pi^2 s\omega^3}{\theta_0^3}$$

$$j\left(\theta=\frac{\theta_0}{4}\right) = \frac{4\pi^2 s\omega^3}{\theta_0^3}\left[\cos\left(\frac{\cancel{2}\pi\cancel{\theta_0}}{\cancel{4}_2\cancel{\theta_0}}\right)\right] = 0$$

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$$j\left(\theta=\frac{\theta_0}{2}\right) = \frac{4\pi^2 S}{\theta_0^3}\omega^3 \cos\left(\frac{2\pi}{\cancel{\theta_0}}\cdot\frac{\cancel{\theta_0}}{2}\right)$$

$$= -\frac{4\pi^2 S\omega^3}{\theta_0^3}.$$

$$j\left(\theta=\frac{3\theta_0}{4}\right) = \frac{4\pi^2 S\omega^3}{\theta_0^3}\cos\left(\frac{2\pi}{\cancel{\theta_0}}\times\frac{3\cancel{\theta_0}}{4_2}\right)$$

$$= \frac{4\pi^2 S\omega^3}{\theta_0^3}\cos\left(\frac{3\pi}{2}\right)$$

$$= 0$$

$$j(\theta=\theta_0) = \frac{4\pi^2 S\omega^3}{\theta_0^3}\cos\left(\frac{2\pi}{\cancel{\theta_0}}\times\cancel{\theta_0}\right)$$

$$= \frac{4\pi^2 S\omega^3}{\theta_0^3}$$

$$\boxed{J_{max} = \frac{4\pi^2 S\omega^3}{\theta_0^3}}$$

Sr no.	Type of motion	Velocity		Acceleration		Jerk	
		$V_o(max)$	$V_R(max)$	$a_o(max.)$	$a_R(max.)$	$J_o(max)$	$J_R(max)$
1	Uniform velocity	$\frac{S}{\theta_0}\omega$	$\frac{S}{\theta_R}\omega$	∞	∞	∞	∞
2.	S.H.M	$\frac{\pi S}{2\theta_0}\omega$	$\frac{\pi S}{2\theta_R}\omega$	$\pm\frac{\pi^2 S}{2\theta_0^2}\omega^2$	$\pm\frac{\pi^2 S}{2\theta_R^2}\omega^2$	$-\frac{\pi^3 S}{2\theta_0^3}\omega^3$	$\frac{\pi^3 S}{2\theta_R^3}\omega^3$
3.	Uniform acceleration & Retardation	$\frac{2S}{\theta_0}\omega$	$-\frac{2S}{\theta_R}\omega$	$\pm\frac{4S}{\theta_0^2}\omega^2$	$\pm\frac{4S}{\theta_R^2}\omega^2$	∞	∞
4	Cycloidal motion	$\frac{2S}{\theta_0}\omega$	$-\frac{2S}{\theta_R}\omega$	$\pm\frac{2\pi S}{\theta_0^2}\omega^2$	$\pm\frac{2\pi S}{\theta_R^2}\omega^2$	$\pm\frac{4\pi^2 S}{\theta_0^3}\omega^3$	$\pm\frac{4\pi^2 S}{\theta_R^3}\omega^3$

10.2 S.H.M motion.

c(11).

$S = 40mm$ at $\theta_0 = \frac{\theta}{4}$ — outstroke $= \frac{360}{4} = 90^\circ$

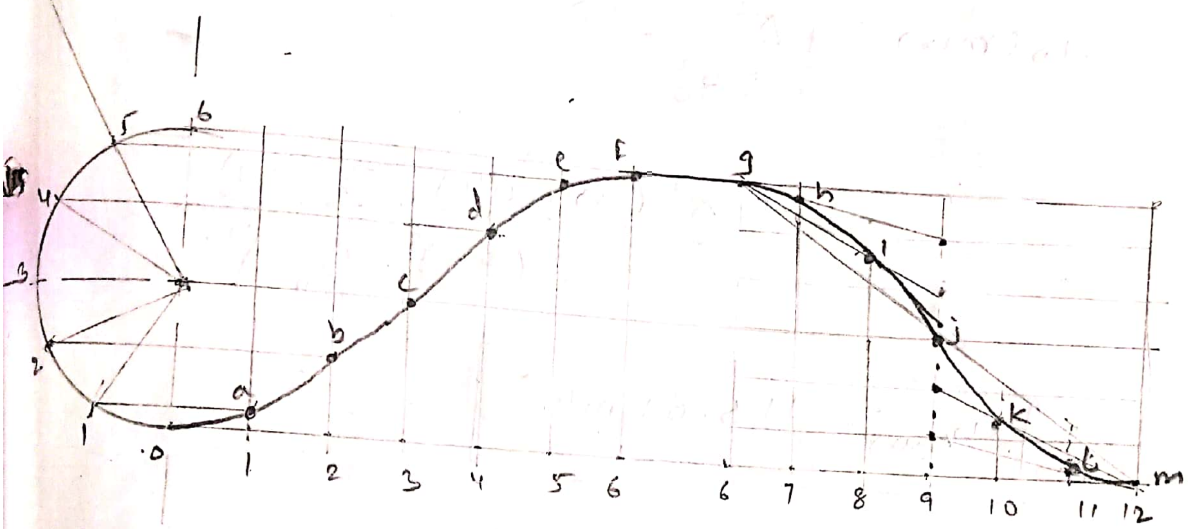
$S = 40$ till $\theta_D = \frac{\theta}{10} = \frac{360}{10} = 36^\circ$

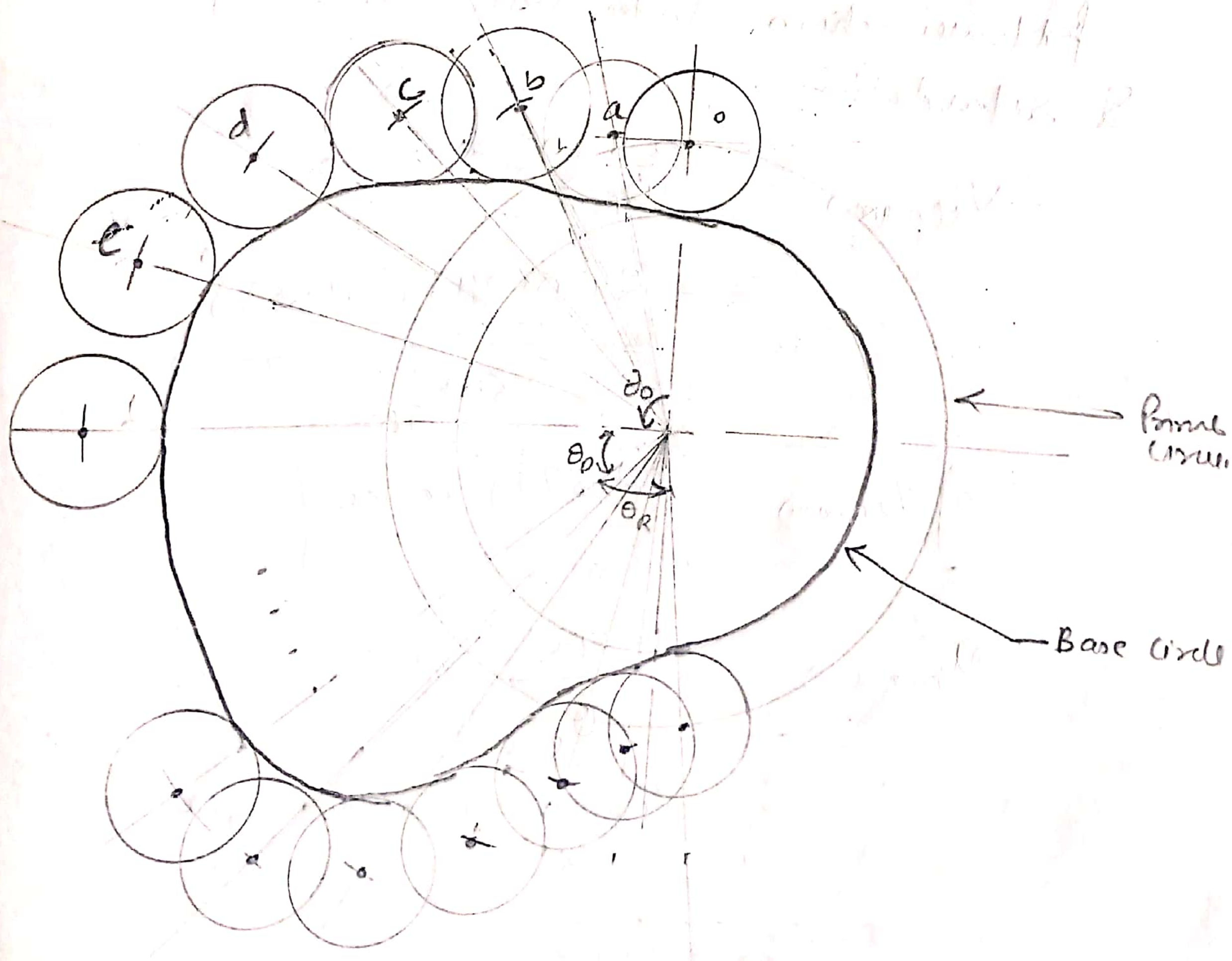
$S = 0$ at $\theta_R = \frac{\theta}{6} = \frac{360}{6} = 60^\circ$

$d_r = 20mm$

$r_r = 30mm$

$N = 360rpm$, $\omega = \frac{2\pi N}{60} = 37.69$ rad/sec.





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Follower ~~moving~~ moving outward with S.H.M.

maxm velocity of follower is,

$$V_{o(max.)} = \frac{\pi S}{2\theta_o}\,\omega$$

$$= \frac{\pi(0.040) \times 37.69}{2(90)\times\frac{\pi}{180}} = 1.508\ m/sec$$

maxm acceleration of follower

$$a_{o(max)} = \pm \frac{\pi^2 S}{2\theta_o^2}\,\omega^2$$

$$= \pm \frac{\pi^2 (0.04)(37.69)^2}{2\left(90\times\frac{\pi}{180}\right)^2}$$

$$\boxed{a_{o(max)} = 113.69\ m/sec}$$

follower return stroke ~~vat~~ with Uniform accel & retardation

$$V_{R(max.)} = -\frac{2S}{\theta_R}\,\omega$$

$$= -\frac{2(0.04)(37.69)}{60\times\frac{\pi}{180}}$$

$$\boxed{V_{R(max)} = -2.8799\ m/sec}$$

$$a_{R(max.)} = \pm\frac{4S}{\theta_R^2}\,\omega^2$$

$$= \frac{4\times(0.04)(37.69)^2}{\left(60\times\frac{\pi}{180}\right)^2}$$

$$\boxed{a_{R(max)} = \pm 207.35\ m/sec^2}$$

10.3

$r_b = 30mm$ $\quad$ $r_r = 10mm$, $\cancel{K_\phi = 0.45mm}$ $\quad$ (12)

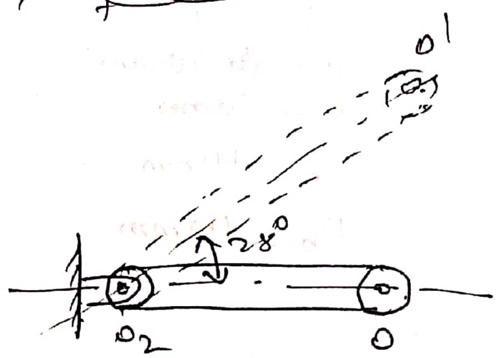
$O_2O = 45mm$, $O_1O_2 = 55mm$

$\theta_o = 80^\circ$

$\theta_R = 120^\circ$

$\theta_D = 50^\circ$

$\alpha = 28^\circ$.



Actual displacement,

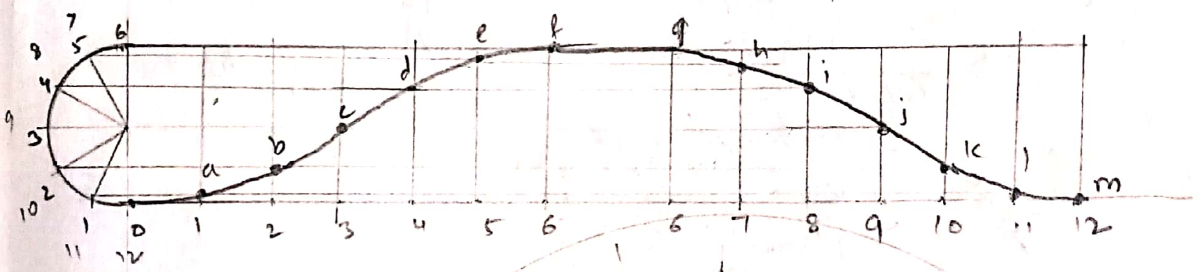
$$S = r\alpha$$

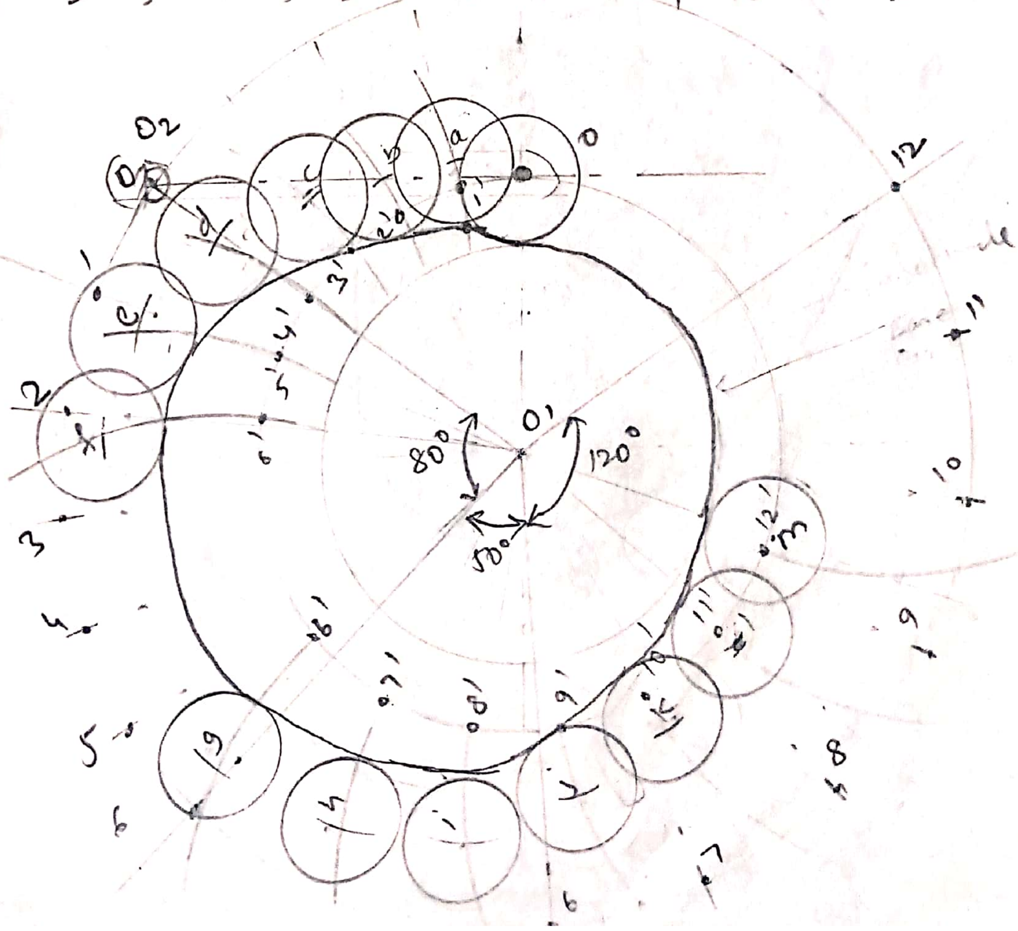
$$= (O_2O)\left(28^\circ \times \frac{\pi}{180}\right)$$

$$= (45)\left(28 \times \frac{\pi}{180}\right)$$

$$\boxed{S = 22mm}$$

Assuming follower moving with S.H.M.





Q. 4. motion of outstroke SHM

$\theta_O = 120^\circ$

$S = 40\,mm$

$e = 10\,mm$

$D_r = 14\,mm$

$D_b = 40\,mm$

motion of return stroke Uniform acct./retar.

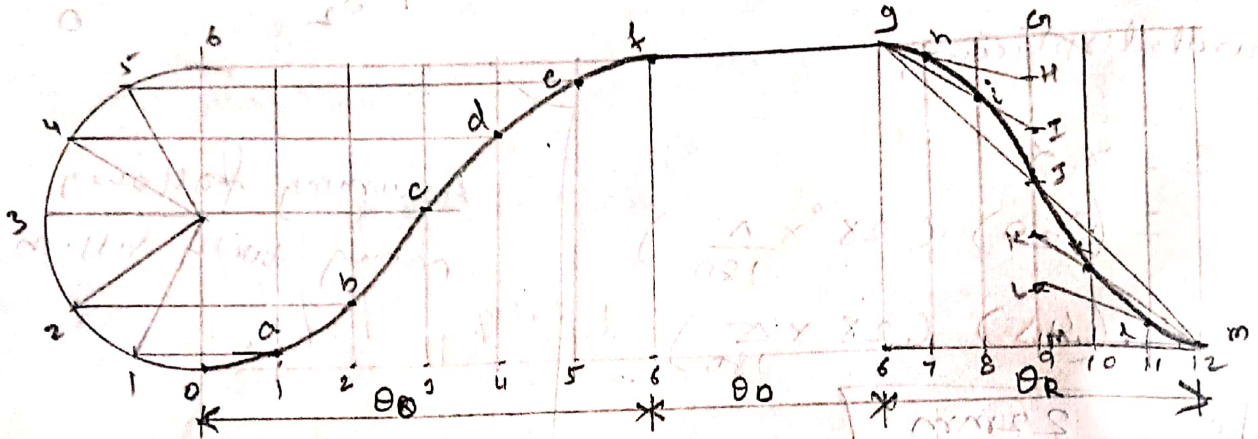
$\theta_R = 80^\circ$

$N = 1000\,rpm$, $\omega = \frac{2\pi N}{60} = 104.71\,rad/sec$

$\theta_D = 60^\circ$

$V_O(max) = ?$ $V_R(max) = ?$

$a_O(max) = ?$ $a_R(max) = ?$



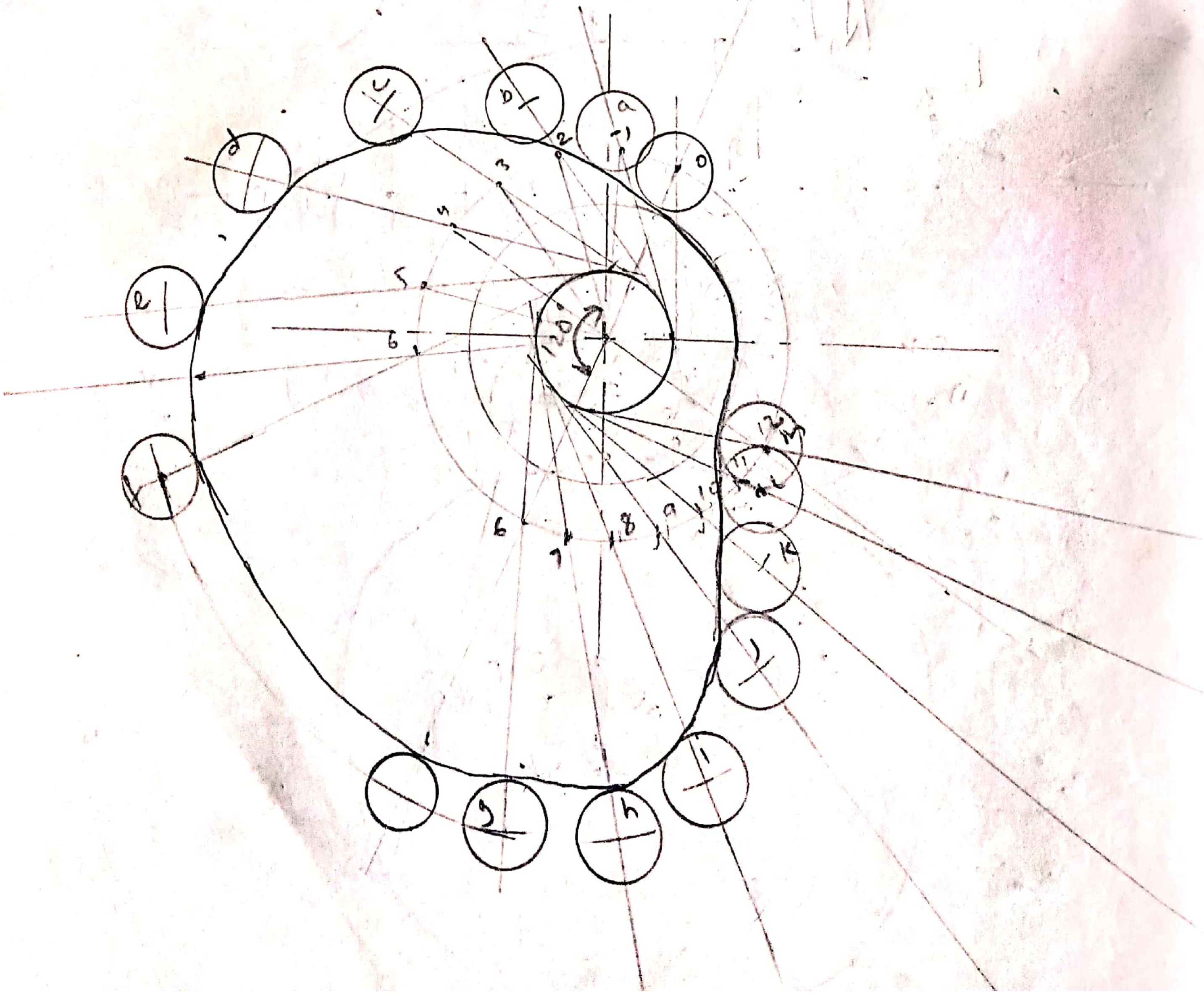
Scale:

$1^\circ = \frac{1}{20}\,cm$

$120^\circ \rightarrow 6\,cm$

$60^\circ \Rightarrow 3\,cm$

$80^\circ \rightarrow 4\,cm$



~~Vcam~~ outward motion is S.H.M

$$V_{o(max.)} = \frac{\pi S}{2\theta_o}\omega = \frac{\pi\cdot(0.04)(104.71)}{2(120\times\frac{\pi}{180})}$$

$$\boxed{V_{o(max)} = 3.14\ m/sec}$$

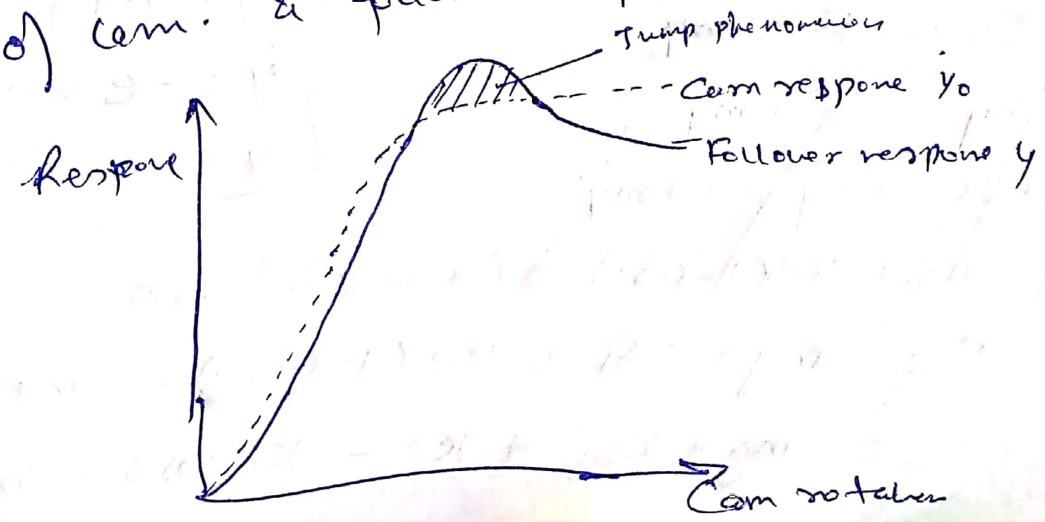
$$a_{o(max)} = \pm\frac{\pi^2 S\cdot\omega^2}{2\theta_o^2} = \pm 493.47\ m/sec^2$$

$$V_{R(max.)} = -\frac{2S}{\theta_R}\omega = -6\ m/sec$$

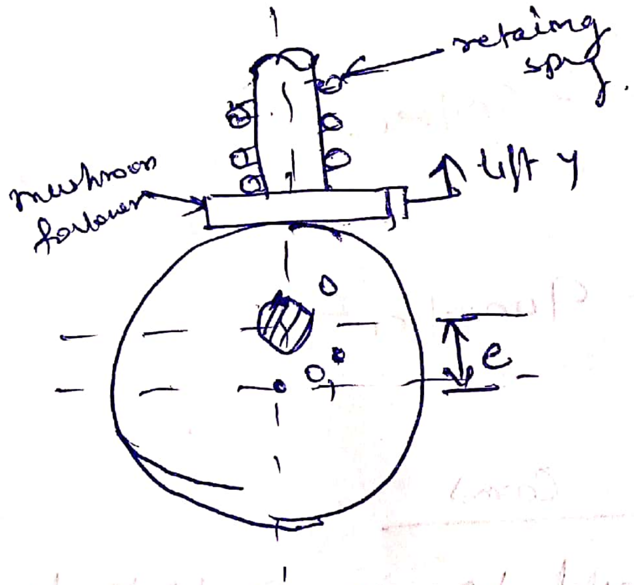
$$a_{R(max.)} = \pm\frac{4\cdot S}{\theta_R^2}\omega^2 = 900\ m/sec^2$$

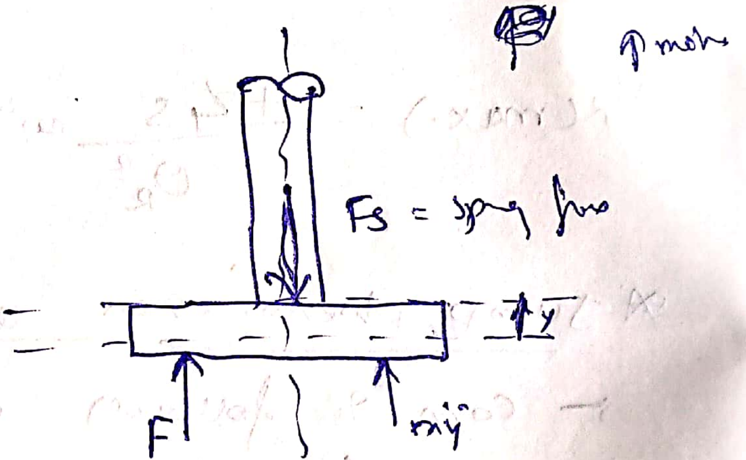
Jump phenomenon in Cams

- Cam & follower should be in contact during working. The retaining spring is used for that.
- Beyond a particular speed of cam, follower may loose contact with the cam due to inertia forces of follower. This phenomenon is called as cam jump.
- occurs due to high speed & flexibility of cam & follower system.



- Jump creates excessively unbalanced forces.
- when follower comes again in contact with the cam surface. It may impact the cam surface, damaging the surface & producing hammering noise.
- cam jump can be avoided by increasing spring stiffness or limiting speed of cam.





F_s = spring force

$F_s = P + Ky$

where

P = preload on spring $= mg + K\delta_i$

where

mg = mass of follower

δ_i = initial spring deflection

F = contact force

$m\ddot{y}$ = inertia force

By D'Alembert principle, inertia force = Σ external

$$P + Ky = F + m\ddot{y}$$

$$mg + K\delta_i + Ky = F + m\ddot{y}$$

For eccentric cam the deflection for particular rotation is given by

$$y = e(1 - \cos\theta)$$

$$\dot{y} = -e\sin\theta \cdot \omega$$

$$\ddot{y} = e\omega^2\cos\theta$$

By D'Alembert's principle

Inertia force = Σ (External force)

$$m\ddot{y} = F - F_s$$

$$m\ddot{y} = F - (P + Ky)$$

$$F = m\ddot{y} + (P + Ky)$$

$\therefore$ $y = e(1 - \cos\theta)$

$$\dot{y} = -e\omega \sin\theta$$

$$\ddot{y} = e\omega^2 \cos\theta$$

$$F = me\omega^2 \cos\theta + P + Ke(1 - \cos\theta)$$

$$= me\omega^2 \cos\theta + P + Ke - Ke\cos\theta$$

$$\boxed{F = P + Ke + (m\omega^2 - K)e\cos\theta}$$

Contact force F is maximum at $\theta = 0°$ i.e $\cos\theta = 1$

" " " minimum " $\theta = 180°$ i.e $\cos\theta = -1$.

when $F \le 0$ or -ve the follower would loose the contact with cam surface which results in jump.

This will happen at particular speed say ω_j

Hence condition becomes

$F = 0$ at $\theta = 180°$

So substituting

$$0 = P + Ke + (m\omega_j^2 - K)e(-1)$$

$$0 = P + Ke - (m\omega_j^2 - K)e$$

$$P + ke = (m\omega_j^2 - K)e$$

$$P + ke = m\omega_j^2 \cdot e - ke$$

$$P + 2ke = m\omega_j^2 \cdot e$$

$$\omega_j^2 = \frac{P + 2ke}{me}$$

$$\boxed{\omega_j = \sqrt{\frac{P + 2ke}{me}}}$$

It means to avoid jump Cam~~g~~ speed

$$\omega < \omega_j$$

$$\omega < \sqrt{\frac{P + 2ke}{me}}$$

Q. $e = 3.75\ cm = 0.0375$

$m = 1.75\ kg$

$K = 24\ N/mm =$ ~~24 N/m~~ ~~0.0024 N/m~~ $24000\ N/m$

$\delta_i = 3.125 = 0.03125\ m$

$\theta = 160°$

$\omega = ?$

max^m usable speed of Cam without jump = ?

~~Compress~~

$$F = P + ke + (m\omega^2 - K)e\cos\theta$$

when jump takes place

$$0 = mg + K\delta_i + ke + (m\omega_j^2 - K)e\cos 100$$

$$0 = (1.75 \times 9.81) + (24000 \times 0.03125) + (24000 \times 0.0375)$$

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$$\boxed{\omega = 400.0189 \text{ rad/sec}}$$

$$\boxed{N = 3819.87 \text{ rpm}}$$

This is the speed at which jump occurs.

* Limiting speed to avoid jump

$$\omega_j = \sqrt{\frac{P + \mu c e}{m e}}$$

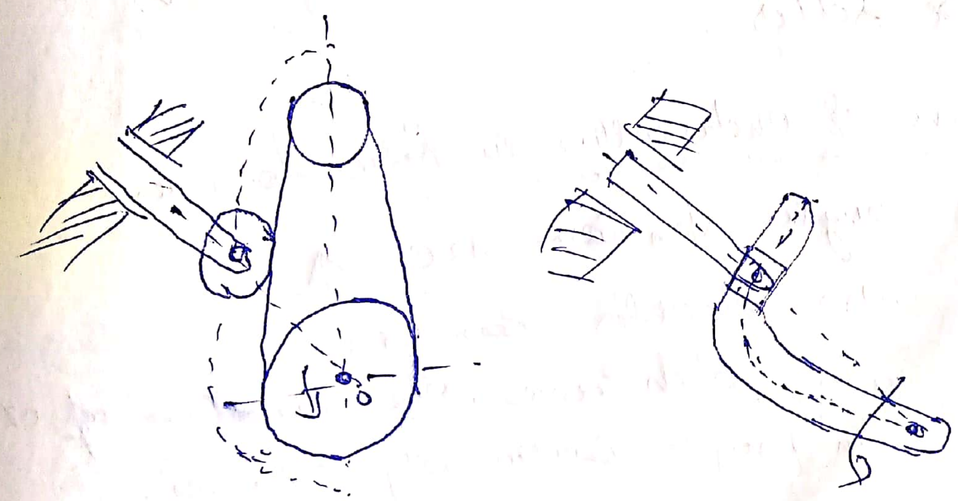
$$= \sqrt{\frac{(1.75 \times 9.81) + (2 \times 240000 \times 0.0375)}{1.75 \times 0.0375}}$$

$$\boxed{\omega_j = 197.784 \text{ rad/sec}}$$

$$\boxed{N_j = 1888.7028 \text{ rpm}}$$

* Kinematically Equivalent System of Cam & follower

By converting into kinematically equivalent system i.e. in a 4-bar mechanism it becomes easy to find the velocity & acceleration of the ~~cam~~ follower easily.



Tangent cam

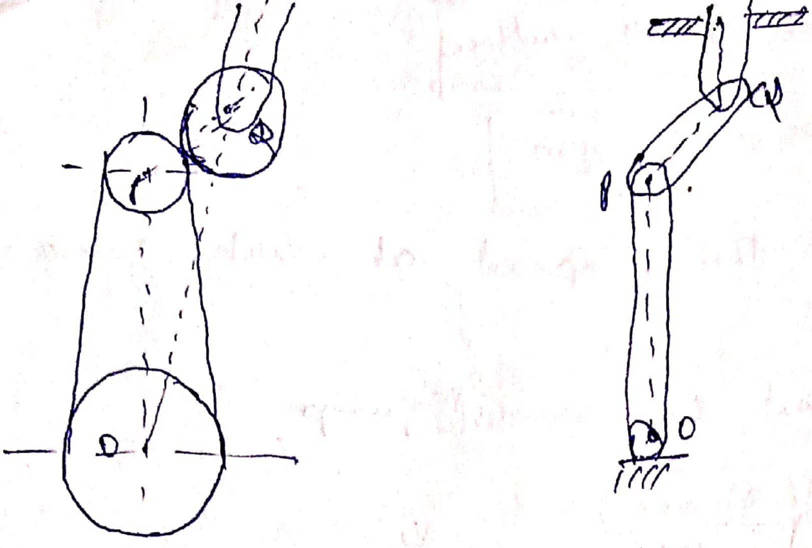


Fig: Tangent cam with roller on the nose

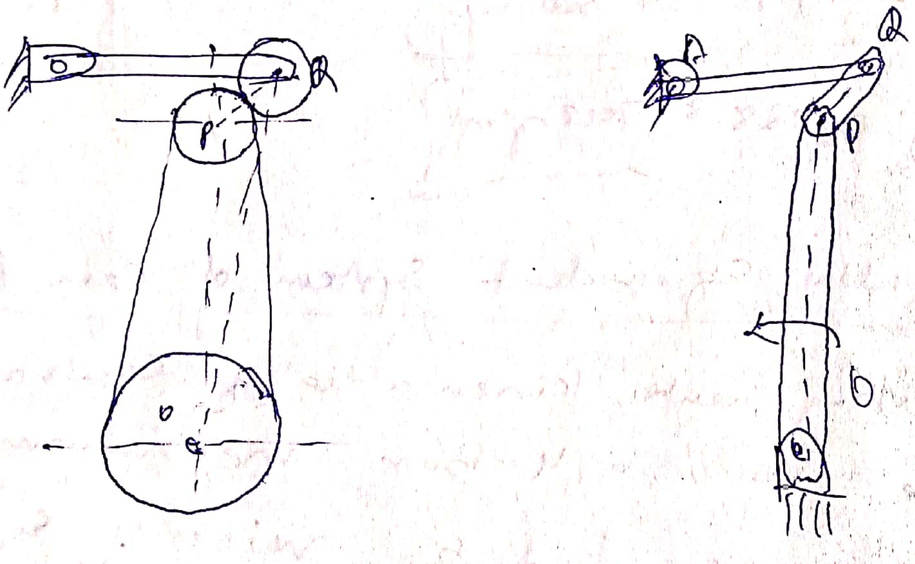


Fig:- Circular arc cam with oscillating follower & roller.

Cam curvature & undercutting in Roller follower

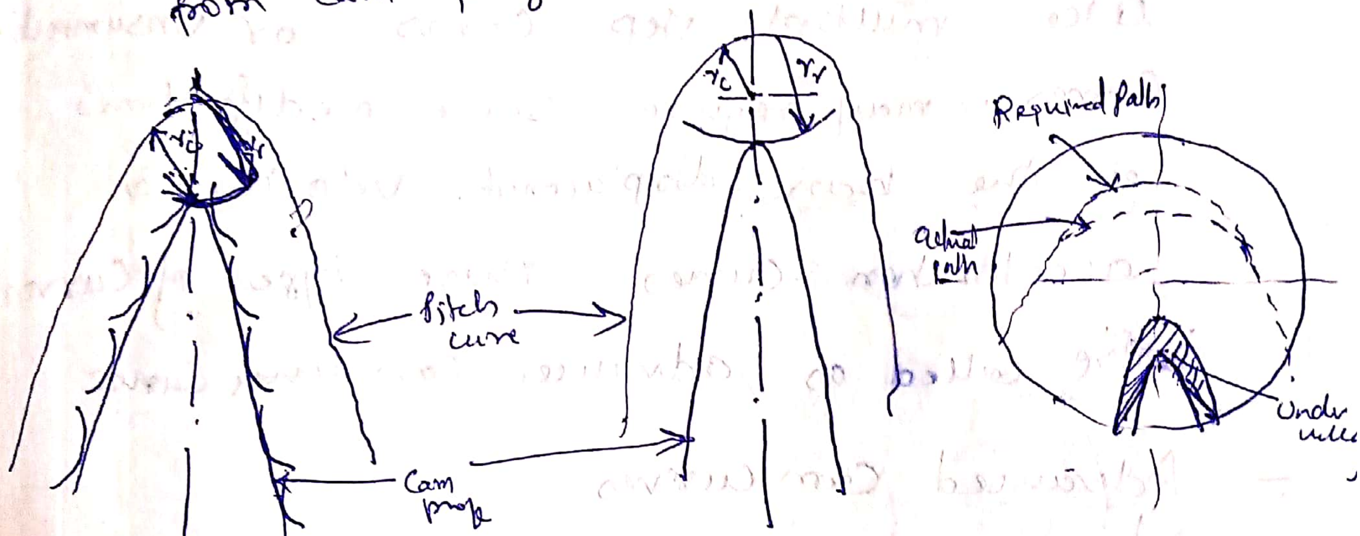
- If Pressure angle ↓, Cam size ↑.
- Cam curvate also decides cam size.
- As we move along pitch curve, the curvature or steepness of cam profile continuously changes.

* Cam Curvature & Undercutting in Roller follower :-

— Pressure angle ↓ Cam size ↑

— Cam curvature is important to decide cam size

- If cam surface curvature is too sharp then follower may not follow prescribed path. This causes very high contact stresses on the surface of cam & by removing some material from cam profile which is called undercutting



$r_c = r_r$

$r_r > r_c$

In this case a sharp point on cam profile creates

In this condition follower doesn't follow the required path of cam profile which will ~~create~~ encounter a condition called Undercutting

- Methods to reduce undercutting
(1) By using smaller roller diameter
(2) By increasing size of cam
(3) By using internal cams.

* Introduction to Advanced Cam Curves

- Simple construction & easy to analyse.

basic cam curves
- Parabolic motion
- SHM
- Cycloidal

- But in certain applications high speed cams or special ~~mat~~ machine motions like multiple step cams or unsymmetric curves may require some modifications in the basic displacement, velocity & acceleration curves. These types of curves are called as advanced cam ~~curve~~ curve

- Advanced cam curves
  - Combining basic curves
  - by numerical procedures
  - by 4th derivative curve
  - by polynomial eqution

In our syllabus we have polynomial eqution curve,

$$y = C_0 + C_1\theta + C_2\theta^2 + C_3\theta^3 + \dots C_n\theta^n$$

where
- y = lift
- θ = Angle of rotation of cam
- $C_0, C_1, C_2, C_3 \dots C_n$ = Constt.

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① Simple polynomial or controlled Displacement polynomial Cam (D-R-D) Cam.

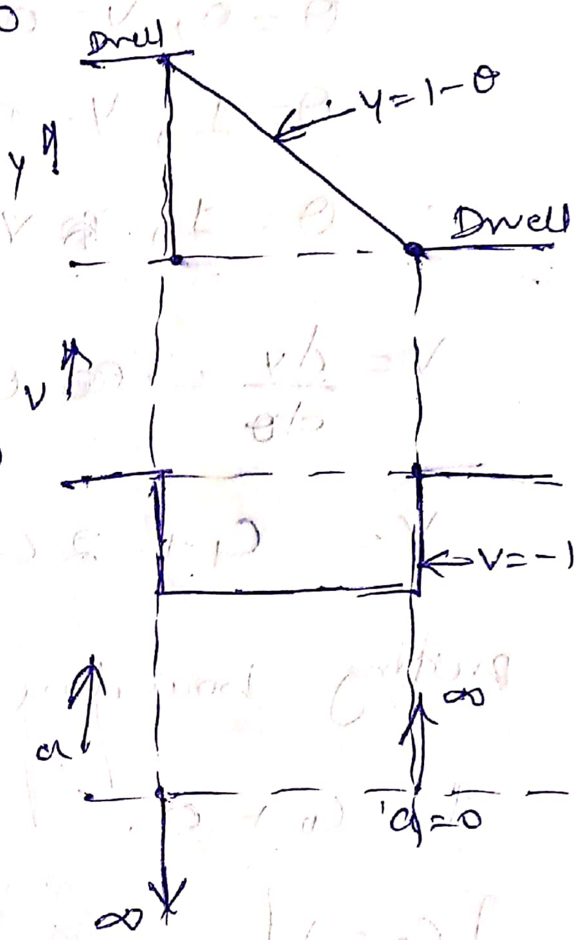
In this case two boundary conditions are considered

$$Y = C_0 + C_1 \theta$$

Boundary conditions are,

at $\theta = 0$, $Y = 1$

at $\theta = 1$, $Y = 0$



hence,

$$Y = C_0 + C_1(0)$$

$$\boxed{C_0 = 1}$$

and,

$$0 = C_0 + C_1(1)$$

$$C_0 + C_1 = 0$$

$$C_0 = -C_1$$

$$\boxed{C_1 = -1}$$

So equation becomes

$$Y = 1 + (-1)\theta$$

$$\boxed{Y = 1 - \theta}$$

$$V = \frac{dy}{d\theta} = -1$$

$$\boxed{V = -1}$$

$$a = \frac{dV}{d\theta} = 0.$$

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(2) 2-3 Polynomial D-R-D Cam

In this Curve 4 boundary Condition are used

$$Y = C_0 + C_1\theta + C_2\theta^2 + C_3\theta^3 \longrightarrow (1)$$

at

$\theta = 0, Y = 1$

$\theta = 0, V = 0$

$\theta = 1, Y = 0$

$\theta = 1, V = 0$

$$V = \frac{dy}{d\theta} = 0 + C_1 + C_2(2\theta) + C_3(3\theta^2)$$

$$V = C_1 + 2C_2\theta + 3C_3\theta^2 \longrightarrow (2)$$

putting boundary Condition in equation (1)

$1 = C_0 + 0.$

$\boxed{C_0 = 1}$

$0 = C_0 + C_1(1) + C_2(1)^2 + C_3(1)^3$

$0 = 1 + C_1 + C_2 + C_3 \longrightarrow (3)$

Putting boundary condition in equation (2)

$\boxed{0 = C_1}$

$0 = C_1 + 2C_2(1) + 3C_3(1)^2$

$0 = C_1 + 2C_2 + 3C_3$

$0 = 2C_2 + 3C_3 \longrightarrow (4)$

Now using equation ③ & ④

$C_2 + C_3 + 1 = 0$

$2C_2 + 3C_3 = 0$

$2[-1 - C_3] + 3C_3 = 0$

$-2 - 2C_3 + 3C_3 = 0$

$-2 + C_3 = 0$

$\boxed{C_3 = 2}$

Putting back C_3 in equation ③

$0 = 1 + 0 + C_2 + 2$

$0 = C_2 + 3$

$\boxed{C_2 = -3}$

So equation becomes

$y = 1 + (0)\theta + (-3)\theta^2 + 2(\theta)^3$

$\boxed{y = 1 - 3\theta^2 + 2\theta^3}$

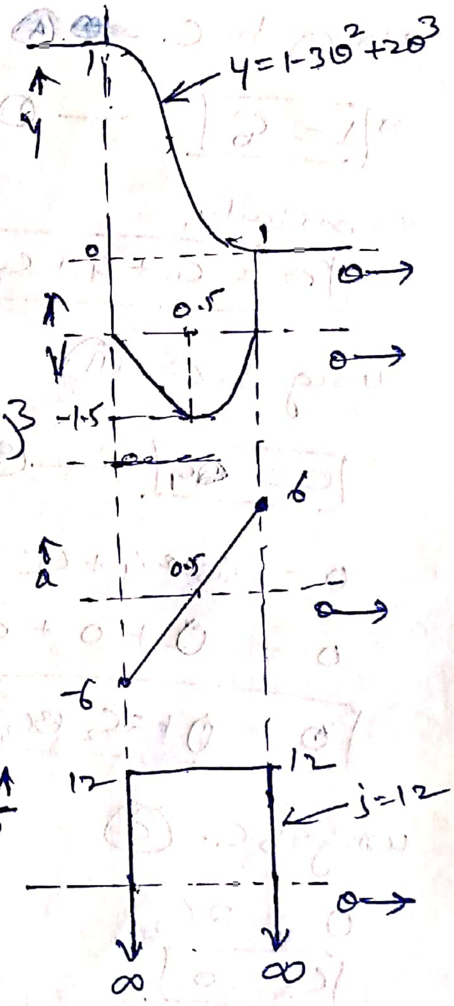
$V = 0 + 2(-3)\theta + 3(2)\theta^2$

$\boxed{V = -6\theta + 6\theta^2}$

$a = \frac{dv}{d\theta} = -6 + 12\theta$

at $\theta = 0$, $a = -6$.

at $\theta = 1$ $a = +6$



$$J = \frac{da}{d\theta} = 12$$

$$\boxed{J = 12}$$

3-4-5 Polynomial D-R-D cam

Six boundary conditions.

$$y = C_0 + C_1\theta + C_2\theta^2 + C_3\theta^3 + C_4\theta^4 + C_5\theta^5$$

at $\theta = 0, y = 1$; $\theta = 1, y = 0$ | at $\theta = 0, V = 0$; $\theta = 1, V = 0$ | $\theta = 0, a = 0$; $\theta = 1, a = 0$

$$V = \frac{dy}{d\theta} = C_1 + 2C_2\theta + 3C_3\theta^2 + 4C_4\theta^3 + 5C_5\theta^4$$

$$a = \frac{dV}{d\theta} = 2C_2 + 6C_3\theta + 12C_4\theta^2 + 20C_5\theta^3$$

using b.C. (A)

$$\boxed{1 = C_0} \quad — (1)$$

similarly,

$$\boxed{0 = C_0 + C_1 + C_2 + C_3 + C_4 + C_5} \quad — (2)$$

or $\boxed{0 = 1 + C_3 + C_4 + C_5}$ — (2)

using b.C. (B)

$$\boxed{0 = C_1} \quad — (3)$$

$$0 = C_1 + C_2 + C_3 + C_4 + C_5$$

$$0 = 0 + 0 + C_2 + C_3 + C_4 + C_5$$

$$\boxed{0 = 0 + 2C_2 + 3C_3 + 4C_4 + 5C_5} \quad — (4)$$

or $\boxed{0 = 3C_3 + 4C_4 + 5C_5}$ — (4)

using b.C. (C)

$$0 = 2C_2$$

$$\boxed{C_2 = 0}$$

$$0 = 2C_2 + 6C_3 + 12C_4 + 20C_5$$

$$\boxed{0 = 6C_3 + 12C_4 + 20C_5} \quad — (5)$$

so, equn are (2) & (4)

$0 = 1 + C_3 + C_4 + C_5$

$0 = 1 + 3C_3 + 4C_4 + 5C_5$

or

$0 = 3 + \cancel{3C_3} + 3C_4 + 3C_5$

$0 = 1 + \cancel{3C_3} + 4C_4 + 5C_5$

$0 = 2 - C_4 - 2C_5$

$\boxed{C_4 + 2C_5 = 2}$ — (6)

similarly solving equtn (2) & (5)

$-6 = \cancel{6C_3} + 6C_4 + 6C_5$

$0 = \cancel{6C_3} + 12C_4 + 20C_5$

$-6 = -6C_4 - 14C_5$

$\boxed{6 = 6C_4 + 14C_5}$ — (7)

~~solving (6) & (7)~~

$\cancel{18 = 6C_4 + 12C_5}$

$\cancel{12 = 6C_4 + 12C_5}$

$\cancel{6 = 2C_5}$

$\cancel{C_5 = 3}$

Solving equtn (6) & (7)

$6 = \cancel{6C_4} + 14C_5$

$18 = \cancel{6C_4} + 12C_5$

$-12 = +2C_5$

$\boxed{C_5 = -6}$

$0 = 1 + C_3 + C_4 + C_5$

$0 = 1 + C_3 + 15 - 6$

$0 = 1 + C_3 + 9$

$C_3 = -10$

$\boxed{C_3 = -10}$

putting C_5 in equtn (6)

$C_4 = 3 - 2C_5$

$= 3 + 12$

$\boxed{C_4 = 15}$

~~putting C_4, C_5 in eqt~~ (2)

$\cancel{C_3 = -1 - C_4 - C_5}$

$\cancel{= -1 - 15 + 3}$

$\cancel{C_3 = -13 = -16 + 3 = -13}$

$$y = c_0 + c_1\theta + c_2\theta^2 + c_3\theta^3 + c_4\theta^4 + c_5\theta^5$$

$$y = 1 + 0 + 0 - 10\theta^3 + 15\theta^4 - 6\theta^5$$

$$y = 1 - 10\theta^3 + 15\theta^4 - 6\theta^5$$

$$v = \frac{dy}{d\theta} = -30\theta^2 + 60\theta^3 - 30\theta^4$$

$$a = \frac{dv}{d\theta} = -60\theta + 180\theta^2 - 120\theta^3$$

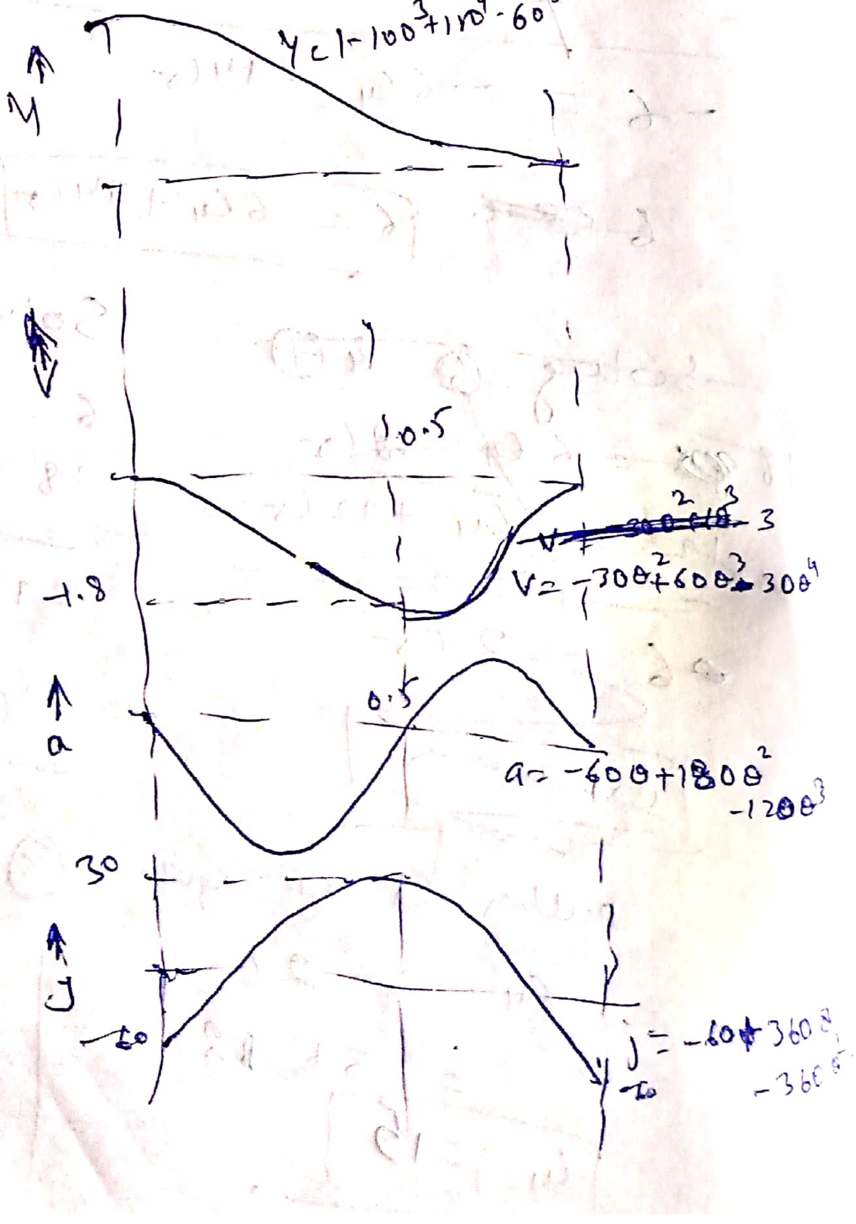
$$J = \frac{da}{d\theta} = -60 + 360\theta - 360\theta^2$$

by calci

$\cancel{c_3 = -10}$

$\cancel{c_4 = 15}$

$\cancel{c_5 = -6}$



$Y = 0.03\,m$ @ $\theta = 90°$

@ $\theta = 0$, $V = 0$

$\theta = 1$, $V = 0$.

$$Y = 1 - 3\theta^2 + 2\theta^3 \quad \text{— (2-3 Cam eqn)}$$

at any instant,

Displacement of follower $= \dfrac{y}{0.03}$

Cam angle $= \dfrac{\theta}{90}$

at any instant.

$$\frac{y}{0.03} = 1 - 3\left(\frac{\theta}{90}\right)^2 + 2\left(\frac{\theta}{90}\right)^3$$

$$y = (0.03) - (0.09)\left(\frac{\theta}{90}\right)^2 + 0.06\left(\frac{\theta}{90}\right)^3$$

$$V = \frac{dy}{d\theta} = 0 - \frac{0.18}{(90)^2}(\theta) + \frac{0.18}{(90)^3}\theta^2$$

$$\boxed{V = -\frac{0.18\,\theta}{90^2} + \frac{0.18}{90^3}\theta^2}$$

$$\boxed{a = \frac{dv}{d\theta} = -\frac{0.18}{90^2} + \frac{0.36}{90^3}\theta}$$

max^m accl^n & velocity.

(1) max^m ~~accl~~ velocity

$$\frac{dv}{d\theta} = 0 = -\frac{0.18}{90^2} + \frac{0.36}{90^3}\theta$$

$$\frac{0.18}{90^2} = \frac{0.36}{90^3}\theta$$

$$\boxed{\theta = 45°}$$

$$V_{max} = -\frac{0.18}{90^2}(45°) + \frac{0.18}{90^3}(45°)$$

$$\boxed{V_{max} = -0.5 \times 10^3\ m/s}$$

(B) max^m $accl^{th}$

~~$\frac{da}{d\theta} = \frac{0.36}{90^3}$~~

$$\therefore\ a = -\frac{0.18}{(90)^2} + \frac{0.36}{(90)^3}\,\theta$$

To get max^m $accl^{th}$ the second term should be large & hence denominator should be small of 2nd term hence

putting $\theta = 90$ will do this

hence at $\theta = 90^\circ$ a is max^m

$$a = -\frac{0.18}{(90)^2} + \frac{0.36 \times 90}{(90)^3}$$

$$= \frac{0.18}{90^2}$$

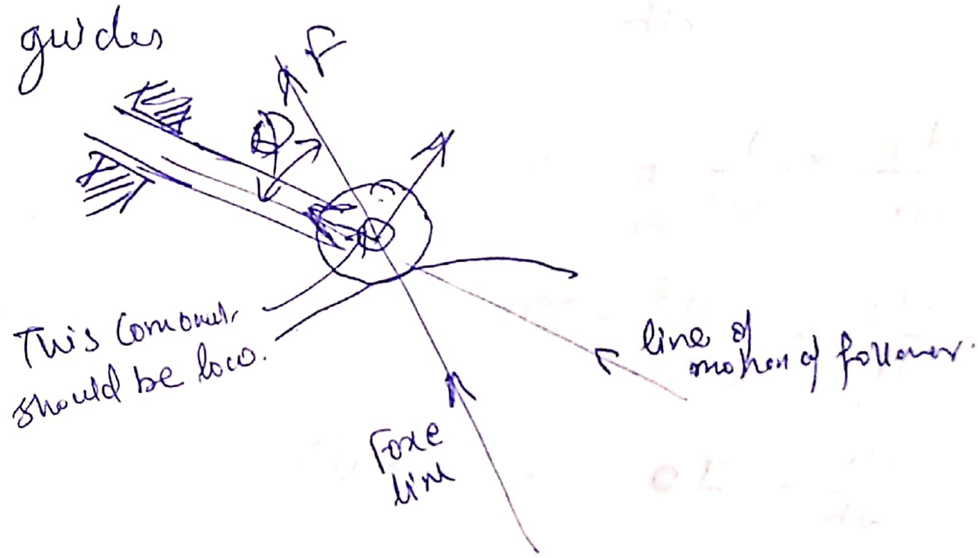
$$\boxed{a = 2.222 \times 10^{-5}\ m/sec^2}$$

* Methods to Control Pressure Angle:-

Pressure angle → Angle between direction of follower motion & a normal to pitch curve.
It is very important in Cam design.

- The o/p load is overcomed by line of force exerted by Cam

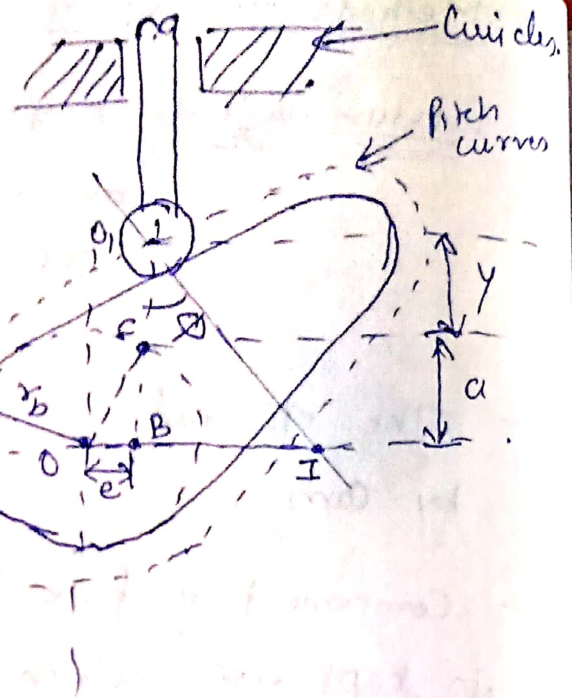
- Component of force along line of follower motion is kept low, which reduces friction between follower & guides



- If value of ϕ is high, → Friction is more in follower & guide. (hence Jam in guide)

ϕ is in betwen (30° to 35°)

I ⇒ Instaneous velocity centre of cam & follower.



$$V_I = \frac{dy}{dt} \quad \text{--- (1)}$$

and

$$V_I = (IO)\,\omega \quad \text{--- (2)}$$

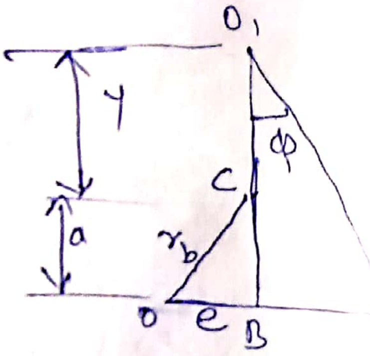
So we can say that from (1) & (2)

$$(IO)\,\omega = \frac{dy}{dt}$$

$$\frac{dy}{dt} \times \frac{1}{\omega} = IO$$

$$\frac{dy}{dt} \times \frac{dt}{d\theta} = IO$$

$$\frac{dy}{d\theta} = IO \quad \text{--- (3)}$$



But $IO = OB + BI$

$\therefore OB = e$

and $BI = (a+y)\tan\phi$

So that equn (3) becomes

$$\frac{dy}{d\theta} = e + (a+y)\tan\phi$$

but $a = \sqrt{r_b^2 - e^2}$

so,

$$\frac{dy}{d\theta} = e + \left[\left(\sqrt{r_b^2 - e^2}\right) + y\right]\tan\phi$$

$$\boxed{\tan\phi = \frac{dy/d\theta - e}{\sqrt{r_b^2 - e^2} + y}}$$

Conclusions

$$\boxed{\tan\phi = \frac{dy/d\theta - e}{\sqrt{r_b^2 - e^2} + y}}$$

(1) if y is known then ϕ can be adjusted by changing e & r_b.

(2) if e is increased, $\phi \downarrow$ (For +ve value of $\frac{dy}{d\theta}$ i.e outstroke,

but $\phi \uparrow$ (For -ve value of $\frac{dy}{d\theta}$) i.e Returnstroke.

generally Forces are greater during rise hence Practically some e is considered to reduce pressure angle.

(3) if $e = 0$

$$\boxed{\tan\phi = \frac{dy/d\theta}{r_b + y}}$$

so by $r_b \uparrow$, $\phi \downarrow$

(4) $\phi \downarrow$ by increasing angle of rotation θ of cam which increase the length of pitch curve for the specified follower displacement. Due to this cam profile become flatter & pressure angle becomes smaller.